NARSIS

New Approach to Reactor Safety ImprovementS

WP4: Applying and comparing various safety assessment approaches on a virtual reactor

Del4.3 – PhD narrative on a model reduction strategy for complex, highly nonlinear and dynamic systems, based on the Proper Generalized Decomposition and LATIN approaches

This project has received funding from the Euratom research and training programme 2014-2018 under Grant Agreement No. 755439.

Project Acronym:	NARSIS
Project Title:	New Approach to Reactor Safety Improvements
Deliverable:	D4.3 - Reactor safety analysis results useful for severe accident analysis, considering deterministic and probabilistic approaches
Month Due:	52
Month delivered:	54
Leading Partner:	CEA
Version:	Final

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Deliverable Review:

Reviewer#1: Giuseppe Rastiello (CEA)

Reviewer#2: Evelyne Foerster (CEA)

Dissemination Level		
PU	Public	**X**
PP	Restricted to other programme participants (including the Commission Services)	
RE	Restricted to a group specified by the consortium (including the Commission Services)	
CO	Confidential, only for members of the consortium (including the Commission Services)	

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1 Executive Summary

1.1 Premise

This document provides a concise summary of the doctoral research entitled:

“Virtual charts for earthquake engineering including loading parameters”

performed by Sebastian Rodriguez-Iturra, PhD at CEA, under the supervision of Dr. Pierre-Etienne Charbonnel (CEA), Prof. Pierre Ladevèze and Prof. David Néron (Ecole Normale Supérieure Paris-Saclay), and Dr. Georges Nahas (IRSN).

The full Ph.D. manuscript is publicly available at the following link:

https://tel.archives-ouvertes.fr/tel-03446766/document

1.2 Summary

The design of civil engineering structures with respect to seismic risk is a very numerically demanding process. This trend is further accentuated when considering nuclear facilities for which additional safety studies are imposed on the most sensitive components. Indeed, the complexity and richness of the numerical models used to predict the often non-linear behaviour of structures generate computation times of several days for the simulation of a single seismic event using classical Newmark-like incremental methods. Furthermore, assessing safety margins and taking into account the variability of the reference problem parameters (mechanical parameters, loading) lead to making this numerical effort, no longer for the simulation of a single model but for a family of models.

This work is dedicated to deriving a strategy for computing parametric solutions, also called numerical charts, for non-linear dynamics in the low-frequency range (typical seismic inputs have a frequency content below 50Hz), with the idea of minimizing the associated computational costs.

Among the different strategies for solving parametric problems, some methods, released in the 2000s and currently booming, propose to use an ingredient referred to as model-order reduction, which confers them a formidable numerical efficiency. The main idea is to exploit information redundancy in the parametric solution to propose an approximated and numerically efficient problem resolution. This guarantees that the calculated approximation, called low-rank approximation, stays close enough to the real solution. The solution of the reference problem is thus approached by a sum of M terms, each of them being a product of functions with separate variables. The integer M is called the rank of the approximation, and, in practice, the approximation space (which basis contains the separate variables functions) is constructed incrementally.

Among other model-order reduction techniques, the Proper Generalized Decomposition (PGD) (Ladevèze, 1999) offers a conducive framework for obtaining parametric solutions in the linear range (see e.g., Ammar et al., 2006; Gunzburger et al., 2007; Chevreuil & Nouy, 2012). In turn, the LATIN (LArge Time INcrement) method proposes a general strategy for resolving non-linear problems in mechanics involving an alternative sequence of non-linear and linear stages (see Figure 1).

At each linear stage, a global space-time problem expressing the equilibrium of the system must be solved. Each non-linear correction $\Delta S^{(n+1)}$ is sought by minimizing a residual integrated on the time T and space Ω domain:

$$\Delta\mathcal{S}^{(n+1)} = \arg\min_{\mathcal{S}\in\mathrm{Ad}} \underbrace{\left\|\Delta\mathcal{S} - \mathcal{S}^{(n)} + \Delta\right\|_{\Omega,T}}_{\mathcal{R}} \tag{1}$$

The PGD is used to provide a reliable and numerically economical low-rank approximation of the solution of this linear problem, by seeking the corrections under the form:

$$\Delta\mathcal{S}^{(n+1)}(t,x) \;=\; \sum_{m=1}^{M} \alpha_m(t)\;\phi_m(x) \tag{2}$$

where the time and space-modes, respectively denotes as α_m and ϕ_m, are computed using a fixed-point strategy with alternate search directions.

In Figure 1, the LATIN/PGD approach is schematically described, showing the non time-incremental iterative strategy. The sought solution S_i associated with a vector θ_i parameterizing the constitutive relations, is at the intersection of a manifold $\boldsymbol{\Gamma}_i$ (on which constitutive relations are verified) and an affine admissibility space $\boldsymbol{Ad}$ (where the kinematic relations and dynamic equilibrium is verified).

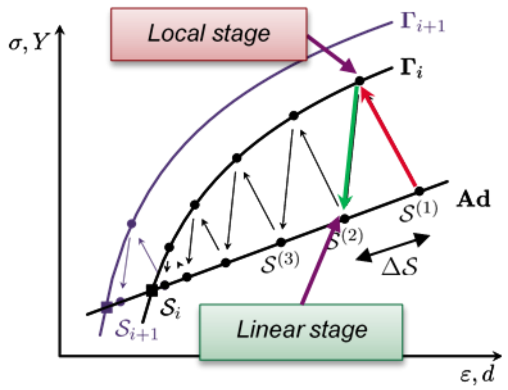

Figure 1 – Overview of the LATIN/PGD approach.

The efficiency of the LATIN/PGD method was already demonstrated on numerous examples, where numerical charts were derived for transient non-linear problems (see e.g., Heyberger et al., 2012; Néron et al., 2015). However, it was never applied to low-frequency dynamic (seismic) problems before the present works. For this purpose, the first steps in applying this methodology to the framework of dynamics were to:

(i) Modify the definition of the admissibility space $\mathbf{Ad}$, by adding an inertia term, which led the residual in eq. (1) to be rewritten;
(ii) Adapt accordingly the fixed-point strategy used to compute the time and space modes of eq. (2).

The derived methodology was successfully tested on several numerical examples involving damaging quasi-brittle (concrete) and elasto-viscoplastic (steel) materials. An example is shown on Figure 2 for a simply supported concrete beam (6m length, 4,535 linear tetrahedrons for finite-element mesh) and submitted to a dynamic motion of its supports. Figure 2 compares the results obtained with the classical step-by-step integration method (Newmark-based time integration and Newton-Raphson algorithm for overall nonlinear solving) and the newly derived LATIN/PGD methodology, using 1000 Lagrange polynomials of order 2 in time for an integration on 20 sec. The damage time histories were obtained at the most loaded Gauss point for both approaches. The whole 3D damage field at the end of both simulations is also shown for the sake of comparison. The results obtained with both methods were in good agreement and the CPU time was slightly (30%) in favour of the LATIN/PGD method. Then, a parameterized solution was computed, by considering a vector θ with constitutive parameters E (Young's modulus), Y_0 (damage activation threshold) and a (loading amplitude), varying in +/- 40% intervals. 1000 LATIN/PGD simulations were run. The solution S_{i+1} for a parameter

θ_{i+1} is initiated to an already converged solution S_i associated with a close parameter set θ_i (Figure 1) to decrease the number of LATIN iterations needed for convergence. Furthermore, the space basis $(\phi_i)_{i=1}^{M}$ is preciously kept and reused throughout the numerical chart computation, making the general methodology particularly efficient. An enrichment stage can be performed if the convergence criterion is not met. Compared to chained classical step-by-step resolution, the computational gain can be estimated to be more than 700% in favour of the LATIN/PGD approach when performing such parametric studies.

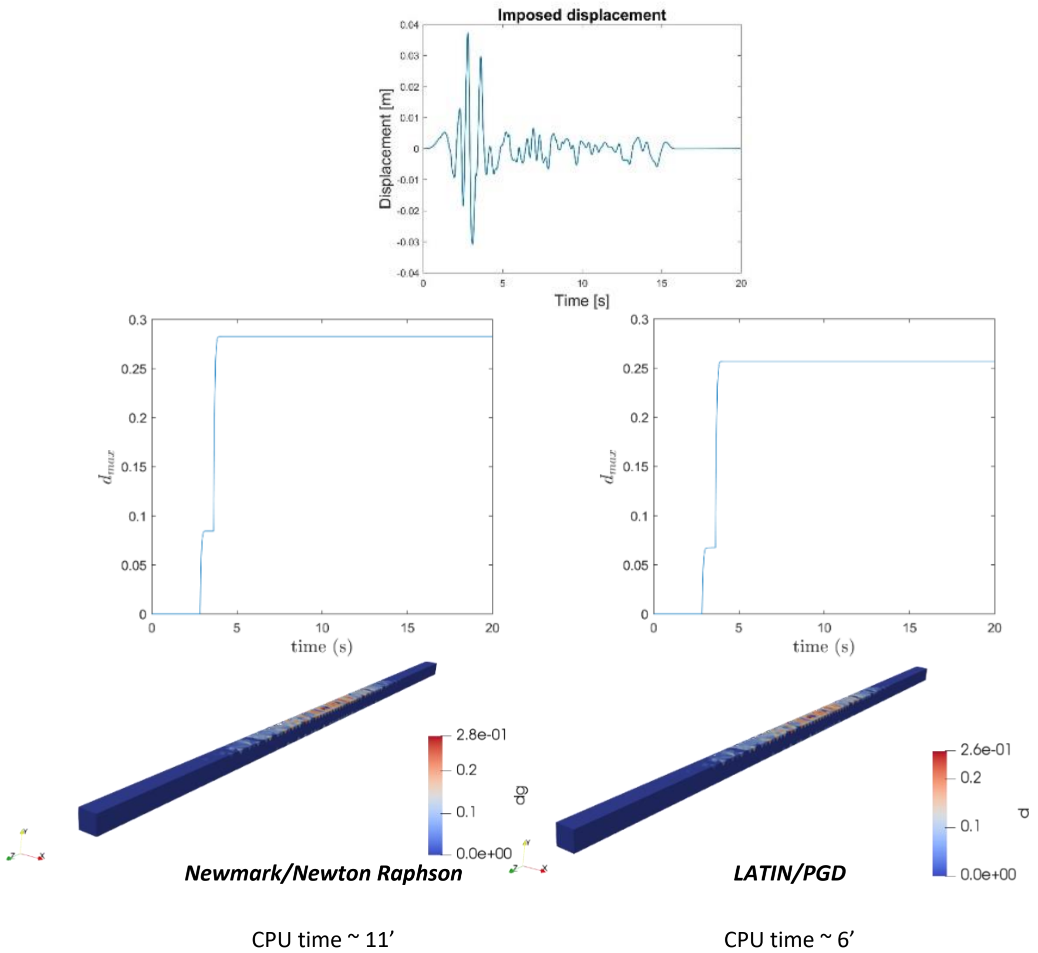

Figure 2 - Comparison between the classical step-by-step and LATIN/PGD resolution in nonlinear dynamics.

Once the parameterized solution is computed, probabilistic analyses can be performed for negligible computational cost by simply interrogating/interpolating the numerical charts. Figure 3 shows an example of deriving some fragility curves quasi-instantaneously from the numerical chart (simple interpolation), considering the beam example previously shown (Figure 2). In this example, the fragility is expressed as the probability of the maximum damage d_{max} to exceed a given damage threshold d_c. In addition, the Young's modulus E and the damage activation threshold Y_0 were supposed to follow a Gaussian distribution.

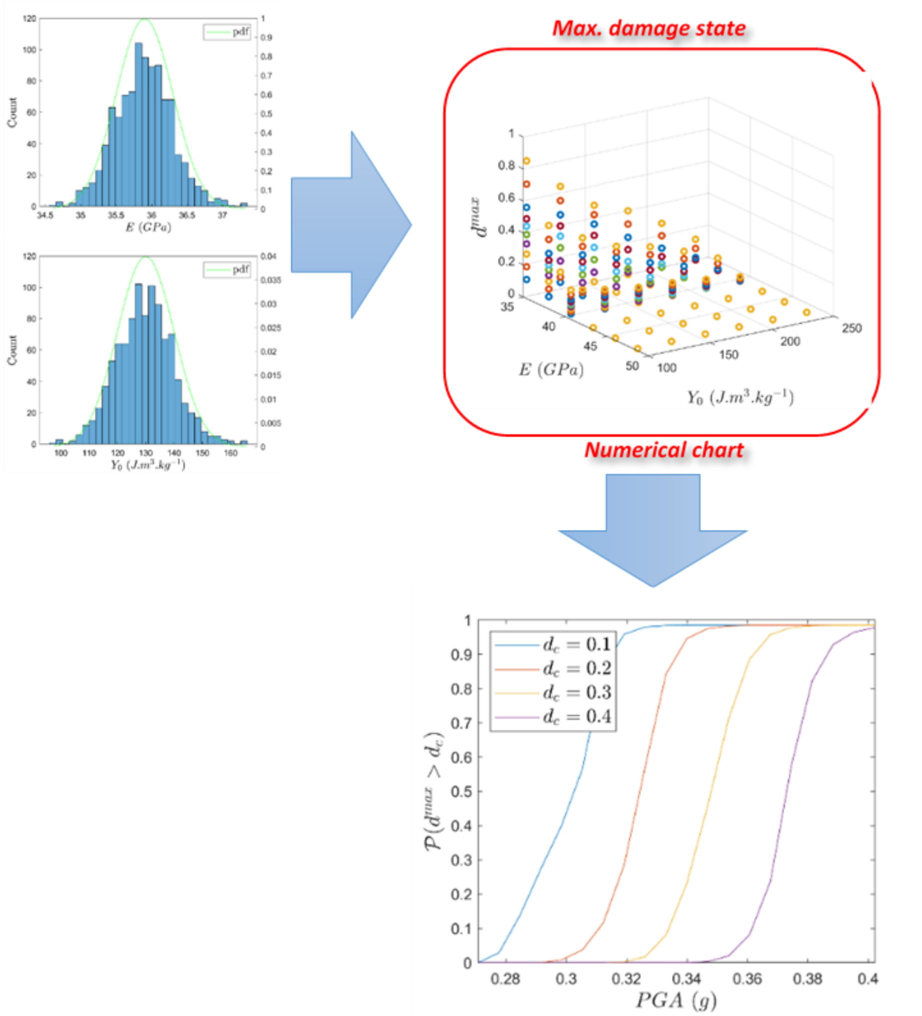

Figure 3 – Example of the use of a parameterized solution (top left) to derive a numerical chart (top right), which are then used to produce fragility curves (bottom right).

The CPU time spent solving the sequence of linear stages within the LATIN/PGD methodology, is however still significant. It can penalize the method's efficiency, especially when considering long seismic events. Hence, some improvements were explored during this PhD works. For instance, a particular effort was made to take advantage of the multi-frequency nature of the seismic input. A new decomposition of the residual *Δ* of eq. (1), which defines the loading for each linear stage, was proposed. It consists in writing *Δ* as a sum of periodic contributions or modes, each mode being associated with a single frequency and an envelope function (Figure 4). A coarse macro time-scale for each mode can then be defined and the correction related to each mode can hence be computed using a reduced number of Gauss points for time integration (performed using 2nd order Lagrange polynomials). The "remeshing" stage in time is quick and automated and enables to decrease the numerical cost associated with time integration and computation of corrections $((\alpha_i)_{i=1}^{M}$ in eq. (2)). This improvement was tested on an elasto-viscoplastic case: it allowed a 38% time saving for the computation of time corrections. However, the computational gain is possible only if the number of modes decomposing the residual Δ, remains reasonably low, which might not be the case for all seismic loadings.

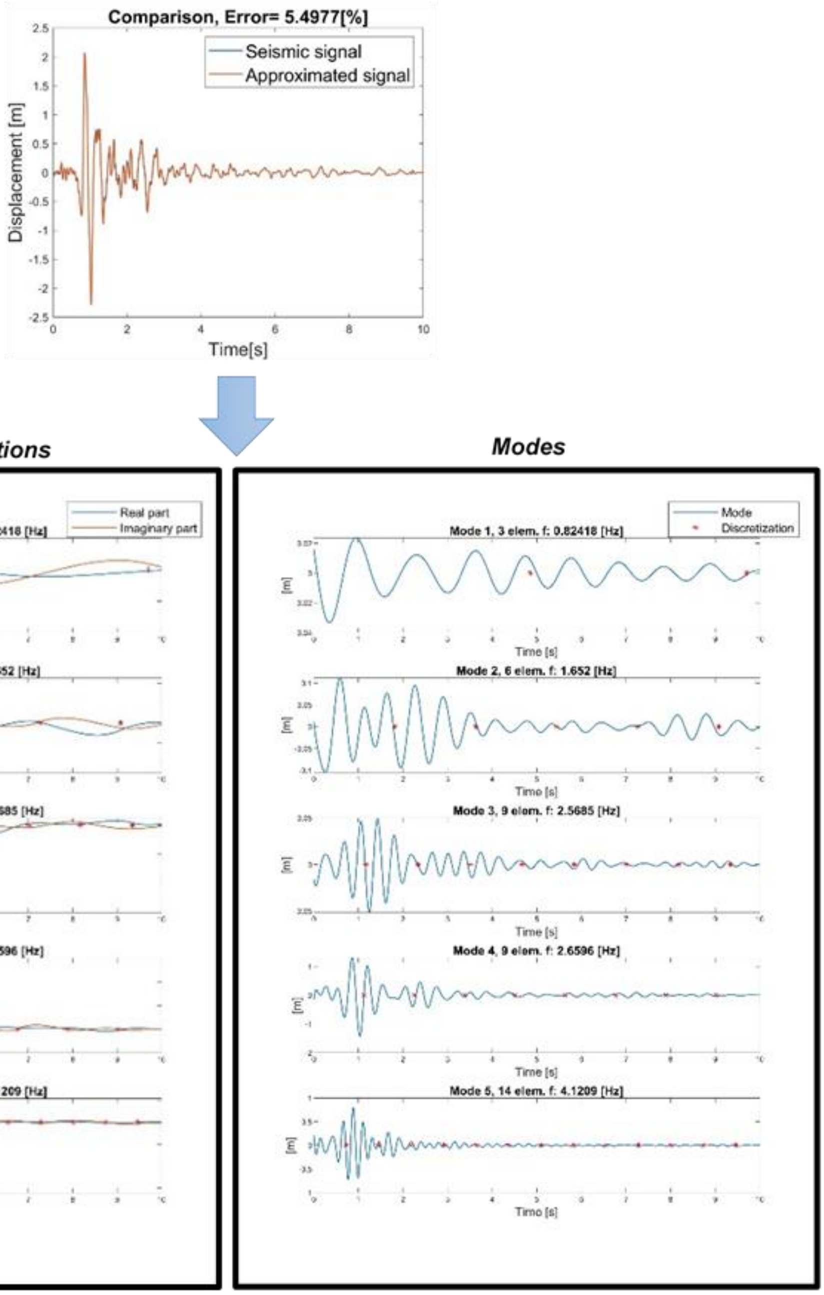

Figure 4 - Decomposition of the residual Δ into periodic contributions (modes): each mode is associated with its frequency and envelope function.

The last development made aimed at improving the nonlinear stage resolution, where the constitutive relations $C(x_i, t_j)$ have to be solved at each Gauss point (j in time and i in space). Once again, a separate variables representation of the constitutive relations on the manifold Γ was sought as follows:

$$C(x,t) \approx \sum_{k=1}^{N} a_k(t)\, b_k(x) \tag{3}$$

In practice, reference points are chosen to characterize the manifold $\boldsymbol{\Gamma}$, and a posteriori Proper Orthogonal Decomposition (POD) is carried out to find the time and space contributions $(a_k, b_k)_{k=1}^{N}$ of the modes in eq. (3). This decomposition can be computed in a preliminary computation stage and once obtained, the resolution of the constitutive relations can be significantly accelerated. An approximated expression of the tangent operators can be obtained from $(a_k, b_k)_{k=1}^{N}$, and they can be used as optimized search directions in the LATIN strategy. This last ingredient was tested for elasto-viscoplastic problems as well, showing encouraging results with up to 50% computation time saving for the nonlinear stages.

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