NARSIS

New Approach to Reactor Safety ImprovementS

WP3: Integration and safety analysis

Del3.7 – Improvement of flexible approaches and procedures relying on expert-based information

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1 Executive Summary

The use of expert-based information is one major pillar of safety analysis. Since the original critiques of the practices in the eighties, a large variety of research studies have been proposed to structure the procedures for the modelling and for the elicitation of expert-based information in situations where empirical data are scarce, incomplete, imprecise and vague. To handle such constraining situations, NARSIS deliverable D3.7 aims at conducting an in-depth applicability and feasibility assessment of flexible approaches / procedures taking advantages (depending on the information available) of recent advances as provided by uncertainty theories that are alternative to “classical” probabilities (more specifically Fuzzy sets and possibility theory).

This has led to the following conclusions and recommendations:

1. Regarding the use of expert-based information in analysis based on Bayesian Networks (BBN),

- The main challenge is the minimization of the elicitation workload on the experts owing to the large number of BBN parameters (Conditional Probability Table CPT entries) while preserving the quality and consistency of the elicited result. A large number of different CPT elicitation procedures exist, but a consensus on the best practices is still lacking. Broader benchmark exercises are here needed to cover a larger spectrum of methods;
- To both fulfil the objective of propagating the uncertainty and conducting the robustness analysis to the expert-based assumptions, the use of imprecise probabilities either relying on interval-valued probabilities within the setting of credal networks or on the Dempster-Shafer theory within the setting of evidential networks can be a valuable tool;
- Though these new approaches allow an increase in expressiveness with respect to uncertainty representation, it should be underlined that this might come at the expense of higher complexity of the inference algorithms (and higher computational costs).

2. Regarding the evaluation of expert-based information, we have investigated the benefit of using an alternative to the classical model of Cooke based on the possibility theory using the interactive web app developed for NARSIS (available here: https://github.com/rohmerj/ExpertScoring):

- Regarding the problem of expert ranking, there are differences between these two approaches, but these can be explained, and searching the reasons for the dissimilarities can improve the evaluation analysis. Being aware that the two approaches provide two different viewpoints on the problem, one may envisage to mix these two pieces of information or leave it to the decision-maker to decide, which assessment best serves her/his particular interests;
- Regarding the problem of prediction, higher accuracy of the predictions appears to be achieved, on the test cases considered, by using the possibilistic approach, but this gain seems to come at the expense of higher imprecision (less informative). Using the classical model is however riskier, because we cannot be certain to find the true values within the interval derived from the averaging aggregation.

3. Regarding the modelling of expert knowledge and reproducing expert-like reasoning, the analysis of a solution based on fuzzy expert systems has highlighted key advantages for both risk assessment, and monitoring and diagnostic analysis of accidents of nuclear plants, namely:

- By combining knowledge and experience in relation to experts, fuzzy expert system allows overcoming the problem of safety data unavailability;

- The use of fuzzy logic allows a more realistic analysis thanks to the approximate reasoning and the use of no crisp information, which avoids threshold effects;
- A higher degree of flexibility can be achieved through the use of linguistic variables, which avoids focusing on precise knowledge and does not need to be interpreted to be used;
- From a practical perspective, since each rule can be edited independently, it is seamless to update the rule base.

To conclude, the tools of new uncertainty theories can be considered valuable ingredients to support the safety analysis, and for NARSIS project in particular. They should however not be seen as supplements to “classical” probabilistic tools, but rather as complements to nuance the results using expert-based information, to put light on different perspectives, and to highlight potential flaws in the assessment process. Given the large variety of decision-making situations, finding a single appropriate framework appears to be debatable, and it is beneficial to take advantages of the strengths of multiple approaches to capture different types of information and knowledge important to inform the decision-making.

2 Introduction

The present report addresses the question of the use of expert-based information for safety analysis of nuclear power plants NPP, in particular when using the BBN-based approach (that is one major pillar of the NARSIS project), and of how to improve the modelling and processing of this particular type of knowledge.

Since the original critiques of the practices in the eighties (e.g. Mosleh et al., 1988), a large variety of research studies have been proposed to structure the process of expert opinion elicitation and use in the situations where empirical data are scarce, incomplete, imprecise and vague, i.e. situations of high degree of uncertainty. The proposed approach will focus on approaches that are alternative to classical probabilities, i.e. non-probabilistic approaches. The rationale for this is described in Sect. 2.1. In Sect. 2.2, we analyse the current practices regarding the use of expert-based information. On this basis, we identify three main research questions with respect to NARSIS objective that are addressed using approaches alternative to classical probabilities (Sect. 2.3).

2.1 Rationale for using non-probabilistic approaches

When dealing with uncertainty, two major facets of uncertainty are generally distinguished as discussed by Hoffman and Hammonds (1994) and Paté-Cornell (1996) and more specifically outlined for performance assessments for complex systems by Helton and Burmaster (1996), and for safety assessments of technological systems by Apostolakis (1990).

- *Aleatory uncertainty/variability* (also referred to as randomness). The physical environment or engineered system under study can behave in different ways or is valued differently spatially or/and temporally. The aleatory variability is associated with the impossibility of predicting deterministically the evolution of a system due to its intrinsic complexity. Hence, this source of uncertainty represents the “real” variability and it is inherent to the physical environment or engineered system under study, i.e., it is an attribute/property. Examples are ocean fluid dynamics and the occurrence of earthquakes;

- *Epistemic uncertainty.* This type is also referred to as “knowledge-based”, as the Greek term episteme means knowledge. In contrast to the first type, epistemic uncertainty is not intrinsic to the system under study and can be qualified as being “artificial”, because it stems from the incomplete/imprecise nature of available information, i.e., the limited knowledge of the physical environment or engineered system under study.

While tools from the probabilistic setting can appropriately handle the first fact, it is the second fact, which raises several problems in practice. The pure-probabilistic representations have been criticized for inducing an appearance of more refined knowledge with respect to the existing uncertainty than is really present (Klir, 1989, Klir, 1994). When experimental data on material properties are insufficient to make a distinction between several probability distributions (e.g., gamma and lognormal), this choice may sound “arbitrary” (see. e.g., Ditlevsen, 1994). O’Hagan and Oakley (2004) argued that the problem does not stem from the use of probability as such, but from the elicitation process (i.e. the procedure consisting in expressing the experts’ knowledge and beliefs in probabilistic form). Nevertheless, in situations of data/information scarcity, expressing information in terms of mean, variance, or any statistical quantities may appear tedious, if not debatable. More specifically, this is supported by the results of some cognitive studies (Raufaste et al., 2003). As outlined by Dubois and Prade (1994), the probability setting may be often too “rich” to be currently supplied by individuals: the identification of the probability distribution requires more information than what an expert is able to supply, which is often restricted to the 0.50 and 0.95 fractiles or a prescribed mode. Given these pieces of information, many mathematical probabilistic laws may exist (Dubois and Prade, 1994). Hence, when the knowledge is restricted to the bounds (like in the afore-described cases), there is an infinity of probabilistic laws defined by such a support.

Recently, alternative non-probabilistic approaches (e.g. possibility theory, imprecise probability e.g. Dubois & Guyonnet, 2011) have been proposed and investigated to represent uncertainty in situations characterized by limited available information, mainly coming from expert judgements. They are named “non-probabilistic”, because alternative representation tools, like intervals, Fuzzy sets, etc.

2.2 Current use of expert-based information for nuclear safety analysis

In using data to support quantitative risk analyses and decision-making, the main principle in the nuclear safety field is that the best applicable data practicably available will be utilized. In terms of reliability and applicability, available data sources may roughly be placed in the following order:

1. Operational data and measurements. This consists of failure data (failure mode, detection time, detection mode e.g. testing or failure during operation, redeployment time etc.), process measurements and so on. Systematic recording of data is routine in the nuclear industry, and available data is commonly processed into parameter collections such as the T-boken (2005) that contains reliability data for various components of boiling water reactors;
2. Experimental data. This covers data from scientific studies, laboratory experiments, testing (e.g. accelerated life testing);
3. Computational data, from e.g. simulation experiments;
4. Design specifications, manufacturer specifications;
5. Expert judgment.

Expert judgment is thus mostly used when data that is both relevant and more reliable is not available and its acquisition is not practicable. However, there are modelling and analysis cases where one or more of the following conditions hold:

- The object of risk assessment is a new design for which applicable empirical data is not available. In this case, using expert judgment requires that there is experience with similar systems, and the underlying physical behaviour and design principles are known (Sims et al., 2008);
- Accurate physical modelling is infeasible, and experimentation is not possible (e.g. highly realistic experimentation in severe accident conditions), would take a very long time (e.g. ageing phenomena of concrete related to final repositories of nuclear waste), or would be prohibitively expensive;
- Experimentation would be impracticable (e.g. rare operator errors that would likely take tens of thousands of simulator runs to occur).
- The system under consideration is of secondary importance to safety, and a rough estimate is thus sufficient.

Within Probabilistic Safety Analysis (PSA), expert judgement is thus needed whenever there is a lack of data or information concerning a modelling issue (or other safety analysis issue), and there is no possibility to collect or produce the data or information in the near future. In addition, expert judgement is needed also if available data or information is inconsistent.

2.2.1 Definition

Expert judgment is “data given by an expert in response to a technical problem” (Meyer and Booker 1990). In nuclear safety, expert opinion is generally sought on two kinds of information:

- Quantitative data such as a failure probability, time when an event takes place, etc.
- Qualitative data, such as dependence structures of systems, validity of assumptions used to construct a model for analysis, or selecting of inputs and outputs to a model. Also an expert’s reasons for giving a particular answer, her interpretation of existing

data, and her answers encoded in qualitative scales (e.g. “poor”, “moderate”, “good”) are qualitative data.

Some variation exists in terminology: sometimes, *expert judgment* is used only about quantitative data, and qualitative data given by an expert is called *expert knowledge* (see, e.g. Sims et al., 2008). In the report, we use the generic term “*expert-based information*” to designate both notions.

Sometimes only a rough assessment is needed, and then any piece of information may be asked directly from a person assumed to be an expert. However, in nuclear safety, it is quite often the case that the assessments need to be accurate and reliable. It is well-documented by psychological research that humans are affected by the context, the framing of the problem, and numerous other factors that may produce bias in the assessments and opinions. To minimize bias from different sources, elicitation methods are used. *Elicitation* is the process of obtaining information from experts in a systematic, structured, scientific manner. Its main objective is to minimize sources of bias and ambiguity to obtain as accurate and reliable assessments as possible. Although all work in nuclear safety may be considered expert work, expert opinion on e.g. model construction or analysis procedures is not usually considered to be within the scope of elicitation. Instead, opinion is elicited from persons that are experts in nuclear systems, structures, components, processes, and human factors.

2.2.2 Role in nuclear safety analyses

Expert opinion has played an important role in nuclear safety analyses ever since the creation of the research field in the 1970s. By the time of the NUREG-1150 study in late 1980s, the methodology of elicitation had matured and elicitation was used systematically in the study (Hora and Iman, 1989). At about the same time, United States Nuclear Regulatory Commission requested two leading experts in expert judgment to write a guide for lay persons (engineers, PSA experts etc.) in elicitation; the guide came out first as a NUREG report (Meyer and Booker, 1990) and then as a book (Meyer and Booker, 1991). The book has been republished by the American Statistical Association and the Society of Industrial and Applied Mathematics in 2001. Some representative examples of work carried out in expert judgements in the nuclear safety field are Vo et al. (1991), Ang and Buttery (1997), and Morton (2009).

The U.S. Nuclear Regulatory Commission (NRC) has published several guide documents concerning expert judgements and expert elicitation related to PSA, other safety analyses and decision-making. The latest expert elicitation guidelines are presented in a white paper by Xing and Morrow (2016). The white paper highlights NUREG-1563 (Kotra et al., 1996), NUREG/CR-6372 (Budnitz et al., 1997) and NUREG-2117 (Kammerer and Ake, 2012) as the guideline documents on which NRC staff have traditionally relied on in expert elicitation. Particularly NUREG/CR-6372 has been often applied in PSA context. Even though it focuses on probabilistic seismic hazard analysis, it includes many good practises that have been considered largely applicable to several other areas of PSA. Anyway, since older documents, like NUREG/CR-6372, focus on specific application areas, the white paper (Xing and Morrow, 2016) provides more general guidance. The white paper discusses various possible elicitation approaches, instead of selecting one recommended approach. NRC has also published many reports on the application of expert judgements in PSA, e.g. NUREG/CR-4550 (core damage frequency from internal events, Wheeler et al., 1989), NUREG/CR-4550 (severe accident risks, Harper et al., 1990) and NUREG-1829 (loss of coolant accident frequencies, Tregoning et al., 2008). Boring et al. (2005) provide simplified guidance for expert elicitation to estimate hardware failure probabilities and human error probabilities.

Expert judgements are needed in several areas of PSA, for example human reliability analysis (HRA), seismic PSA, common cause failure analysis, PSA of digital automation systems and level 2 PSA. The information that is needed is typically a probability of an event or the value of a parameter used in the computation a probability of an event, but can also be a consequence of an event, choice of a computation model or decision to apply specific data. Boring et al. (2005) point out that expert elicitation should be applied only to risk significant cases, because a formal expert elicitation process is time-consuming and expensive. Therefore, most expert judgements are actually performed informally (Boring, 2015).

The formats of expert judgements depend on what is being judged.

- If a value for a continuous parameter, like probability, is being evaluated, experts are typically asked to provide information about the distribution of their belief on the parameter value. For example, 5th, 50th and 95th percentile values may be specified. The low and high percentile values that are asked from experts vary;
- If the parameter is a probability, it may not be necessary to ask the low (e.g. 5th) percentile as guided in (Boring et al., 2005). Justification, assumptions and additional specifications may be asked along with the values (Xing et al., 2019);
- For other parameters than probabilities, there may be need to judge aleatory uncertainties and epistemic uncertainties separately as guided in NUREG/CR-6372 (Budnitz et al., 1997). Complete distributions are usually not asked, but that may also be done in some cases. Sometimes experts have had freedom to choose in which form they specify their judgements, i.e. complete distributions or selected percentile values (Harper et al., 1990). On the other hand, sometimes expert judgements are qualitative information that is used to support quantification. For example, in HRA, performance shaping factors are typically determined based on qualitative assessment of different aspects related to human actions. If the experts do not deal with probabilities in their work, one approach is to formulate a set of true/false questions to determine the probability value (Boring, 2015).

Regarding the use of non-probabilistic tools for the (mathematical) representation processing of expert-based information, NRC, to our best knowledge, does not have a specific standpoint: no significant references to fuzzy in the more influential NRC documents. Probability theory appears to be the main approach for dealing with uncertainty that NRC favours. This is however not to say that fuzzy sets (or any other non-probabilistic approaches) would not have received any attention in the nuclear field. The google search 'fuzzy site:inis.iaea.org'[1] brings 3540 results. Although some of them refer to the dictionary meaning of fuzzy rather than the technical term, the number testifies that there has been a lot of research activity related to the application of fuzzy sets, logic and systems in the nuclear domain. The question of how much of this activity has gone to engineering problems and how much to safety-related issues should be deepened.

2.2.3 Key questions

Experts in nuclear safety analysis have identified some key questions that merit further research:

On the use of tools of the probabilistic setting:

- A challenge is to get probability estimates from experts that do not have experience on probabilistic analysis (Boring, 2015). Judgement of uncertainties being particularly challenging;
- It is sometimes challenging to decide the form of the probability distribution to be estimated based on the expert judgements (Subudhi and Martinez-Guridi, 2014).

On the elicitation procedure:

- There is no clear guidance on how to weight the inputs from different experts, if the experts are not equally qualified to judge the subject of assessment (Xing and Morrow, 2016).
- Analysts using the expert elicitation have usually not been trained for expert elicitation, which makes it difficult to select correct methods (Boring, 2015).
- Analysts interviewed in (Boring, 2015) feel that expert elicitation leads often to too conservative estimates.

[1] INIS is IAEA's database of nuclear-related research

- Full scope expert elicitation process is also sometimes seen too time-consuming, and there is a need to simplify the process.
- The expertise of the experts does not always completely match the subject of assessment, and the time available for the judgements is often very limited (Boring, 2015).

2.3 Objectives w.r.t NARSIS project

The BBN-based approach is one major pillar of the NARSIS project since it enables the practitioners to integrate the important contributions of technical (physical and spatial effects), social and human factors into a full picture of threats in order to design an adequate resilience, i.e. mathematically represent any type of knowledge and eventually to support decision under uncertainty.

Yet, in constraining situations (where empirical data are scarce, incomplete, imprecise and vague), the BBN construction and use is largely based on expert-based information and a comprehensive analysis of how such information affects the BBN-based results is needed. This is the purpose of Sect. 3. In particular, Sect. 3 addresses the question of applicability and feasibility of flexible new approaches / procedures taking advantages (depending on the information available) of new uncertainty theories.

One option to constrain the uncertainties related to expert-based information is to combine the opinions of a panel of experts i.e. to combine several sources of information. Yet, a necessary preliminary step is to assess the "quality" of these sources of information, i.e. this raises the practical question of the evaluation of expert knowledge. In Sect. 4, we explore the applicability of procedures described by Destercke & Chojnacki (2008) based on the concept of informativeness and of calibration in the framework of a new uncertainty theory, namely the possibility theory.

The third objective addresses the problem of expert-based knowledge modelling to support decision making. A solution based on fuzzy logic expert system tools is here investigated (Sect. 5).

3 Role of expert-based information in BBN-based analysis

In this section, we analyse how expert-based information is used in the method for safety analysis selected in NARSIS project, namely Bayesian Belief Network (BBN). In the following, we first provide further details on BBN and more specifically formulate the problem of uncertainties in such type of method with a specific emphasis on expert-based information. In Section 3.2, we highlight the difficulty in populating the BBN parameters only based on data/observations. Thus, the first part highlights the necessity for overcoming the lack of data by complementing with additional sources of information. Sect. 3.3 then explores the expert-based option for constraining the uncertainties. Sect. 3.4 provides an overview of the different approaches embedded in different uncertainty analysis settings for evaluating the impacts of such uncertainties, either using probabilities or imprecise probabilities. Sect. 3.5 further addresses the problem of screening these uncertainties by describing sensitivity analysis techniques. Finally, Sect. 3.6 summarizes the main findings and discusses the open questions.

3.1 Context and objectives

Bayesian Belief Network (BBN) has become an increasingly popular method for the analysis of complex systems in various domains of application, like ecosystems (Milns et al., 2010), genetics and biology (Scutari et al., 2014), agriculture (Drury et al., 2017), industry (Weber et al., 2012), financial forecasting (Malagrino et al., 2018), marine safety (Hänninen et al., 2014), human reliability assessment (Mkrtchyan et al., 2015), nuclear power plants (Kwag and Gupta, 2017), aviation risk analysis (Brooker, 2011), coastal systems (Jäger et al., 2018), structure reliability assessments (Langseth and Portinale, 2007), multi-hazard risk assessments (Gehl and D'Ayala, 2016), etc.

Its benefits are: (1) its high flexibility to model any causal relationships; (2) its capability to integrate information from any kind of sources, including experimental data, historical data, and prior expert opinion, and (3) its capability to answer probabilistic queries about them and to find out updated knowledge of the state of a subset of variables when other variables (i.e. the evidence variables) are observed.

Formally, a Bayesian belief Network (BBN) is a class of graphical model (see Jensen, 2001 for a complete and detailed introduction to BBNs), which allows to synthetically represent relations among random variables by means of a directed acyclic graph (DAG) composed of nodes (i.e. the states of the random variables) and arcs (i.e. dependency between nodes). The value of the nodes may be discrete or continuous, and we focus here on the former case, which is the most widely used. For instance, a Boolean node representing the state of a system component can be either "True" or "False". The nodes connected by an arc are called the parent nodes and child nodes respectively. One child node may have several parent nodes, meaning that this node is affected by several factors. Similarly, a parent node could have several child nodes, meaning that this factor may have influences on several other factors. Conditional probabilities are the probabilities that reflect the degree of influence of the parent nodes on the child node. For BBNs with discrete nodes, the probabilistic dependence (i.e. the cause-effect relation) is often represented via a table called a Conditional Probability Table (CPT).

As an illustration, Fig. 1 depicts the binary BBN adapted by van der Gaag et al. (2013) from Cooper (1984) in the field of oncology. The network is composed of 6 nodes and 6 arcs. Node MC refers to metastatic cancer, which may potentially lead to the development of a brain tumor (node B) and may give rise to an increased level of serum calcium (node ISC). The presence of a brain tumour can be established from a CT scan (node CT). Another indicator of the presence of a brain tumour can be related to severe headaches (node SH). A brain tumour or an increased level of serum calcium are both likely to cause a patient to fall into a coma (the node C is connected to node B and node ISC). The conditional probabilistic relationships between the nodes (CPT entries) are provided in Fig. 1 next to the corresponding nodes. For instance, the probability that a patient falls into coma given brain tumor and increased level of serum calcium corresponds to the first entry of the table (1st row, 1st column), namely P(C=True|B=True,ISC=True)=0.80.

Two key ingredients are necessary to build a BBN, namely (1) the graph structure with the direction of the arcs, i.e. the DAG; (2) the states of nodes and the strength of the relationships between nodes, i.e. the CPT. In the present study, we assume that the DAG model has already been determined and restrict the analysis to the quantification of the BBN relationships. The process of deriving the CPTs and its associated uncertainties is recognized in the literature as one of the most delicate part of the BBN development (e.g., Chen and Pollino, 2012; Druzdzel and van der Gaag, 2000; Marcot et al., 2006; Cain, 2001, etc.). It should, however, be noted that the process of DAG derivation (i.e. building the graph structure plus the directions; also known as causal structure learning) has its own challenges as well, in particular when the learning is based on data (see e.g., a comprehensive review by Heinze-Deml et al., 2018).

Depending on the available data (observations, prior knowledge, expert-based information, etc.), the BBN parameters, i.e. the entries of the conditional probability tables (denoted CPTs) can be estimated in different manners, i.e. different assumptions can be made and different methods are available. This means that the results when using the BBN (i.e. uncertainty propagation or probability queries) can be highly sensitive to these choices/assumptions. For NARSIS project, this means that any risk analysis performed using the WP3-based BBN in WP4 and WP5 can be uncertain to the BBN parametrization, i.e. the choices in the CPT values.

This raises the following questions: (1) how to constrain the uncertainties related to CPT derivation, i.e. what are the methods that are available to minimize these uncertainties? (2) how to integrate these uncertainties in the BBN-based analysis, i.e. what are the methods for propagating these uncertainties? (3) how to test the robustness of the BBN-based results to these uncertainties, i.e. what are the methods for identifying the most influential uncertainties? These questions are addressed below through an extensive review of studies performed in the past ten years by focusing on discrete BBNs that can be used for modelling complex causal relationships, for merging different information sources, for prediction, and for belief/evidence propagation (i.e. probabilistic queries). Continuous BBNs (i.e. BBNs with continuous nodes) and dynamic BBNs (i.e. BBNs adapted to model systems evolving over time) are out of the scope of the review.

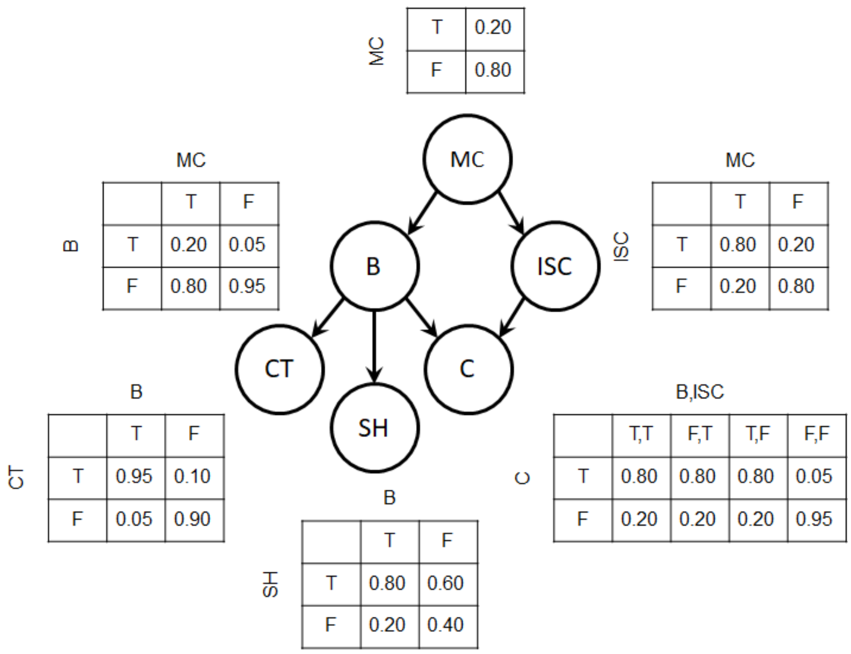

Figure 1: Binary BBN adapted by van der Gaag et al. (2013) from Cooper (1984) in the field of oncology. The tables (called CPT) next to the nodes provide the conditional probabilities values.

3.2 Learning CPT from data

In this section, we address the issues related to deriving the CPT entries from data. Sect. 3.2.1 first discusses the problem of performing this task by using only data. Sect 3.2.2 and 3.2.3 further discuss two practical difficulties, namely: (1) the presence of missing values and (2) the problem of translating observations related to continuous variables into a limited number of discrete states. Finally, Sect. 3.2.4 describes methods that make the most out of scarce data while exploiting qualitative information provided by experts.

3.2.1 A pure data-driven approach

Let us consider a BBN composed of n discrete nodes $X_{i=1,\ldots,n}$. Let us denote r_i the cardinality of X_i (i.e. the number of levels that X_i can take) and q_i the one of the set of parent nodes of X_i, denoted pa(X_i). The k[th] probability value of the conditional probability distribution is $\theta_{\mathrm{ijk}} = P(X_i = k|pa(X_i) = j)$ where i=1,…,n; j=1,….,q_i; k=1,…., r_i.

In data rich contexts, CPT parameters can be evaluated by computing the appropriate frequencies from data. An example is provided by Chojnacki et al. (2019) for fire safety analysis where more than 1 million of numerical simulation results are used. This method corresponds to the maximum likelihood estimation (MLE), which is described below.

Let us consider a dataset D where a total number N_{ij} of data records are available for which pa(X_i) is in the state j and where N_{ijk} data records are available for which X_i is in the state k and pa(X_i) is in the state j. MLE aims at maximizing the log-likelihood function $l(.)$ of θ given D as follows

$$l(\theta|D) = \log\big(P(D|\theta)\big) = \textstyle\sum_{\mathrm{ijk}} N_{\mathrm{ijk}} log\,(\theta_{\mathrm{ijk}}) \tag{1}$$

The solution is then $\frac{N_{\mathrm{ijk}}}{N_{\mathrm{ij}}}$.

The MLE method however fails to find good estimates due to data scarcity when $N_{\mathrm{ij}}\approx 0$, i.e. when training data are not sufficient in number in some specific variable state configurations. Examples of such contexts are not rare in practice; see e.g. rare disease diagnostic (Seixas et al., 2014), accident prevention (e.g., Hänninen, 2014), reliability analysis (e.g., Musharraf et al., 2014), etc. This problem is even worsened when the number of nodes increases. Recall that the number of conditional probabilities is exponential with the number of its parent nodes, i.e. for a node with *i* states and *k* parent nodes and if each parent node has *n* states, $(i-1)\times n^k$ CPT entry values have to be specified. For instance, a binary node with 2 binary parent nodes imposes to specify 4 entries, whereas for a ternary node with 2 ternary nodes, this number reaches 18.

3.2.2 Dealing with missing values

The process for parameter learning of discrete BBNs may be complicated in the presence of missing values. This can be handled by means of different algorithms. The most popular ones are Expectation Maximization (Dempster et al., 1977) and Gibbs sampling (Geman and Geman, 1984). Yet, they both assume that the values are missing at random. This hypothesis may not always be true in practice. Alternative methods have been proposed to overcome this disadvantage, like AI&M procedure (Jaeger, 2006), the RBE algorithm (Ramoni and Sebastiani, 2001), and the maximum entropy method (Cowell, 1999). Other methods have also been developed to speed up the learning process, like generalized conjugate gradient algorithm by Thiesson (1995) or the online updating of rules (Bauer et al., 1997). To deal with both missing data and qualitative influences (as described in Sect. 3.2.4), some initiatives have been proposed like the one of Masegosa et al. (2016), who further improved the combined Isotonic Regression - EM approach.

3.2.3 Discretising continuous variables

A second practical difficulty for parameter learning of discrete BBNs is inherent to the main assumption introduced by discrete BBNs, namely that data should be represented by a limited numbers of outcomes. This imposes to discretize continuous variables. This process might, however, lead to a loss of information, and potentially to an increase of the associated computational effort, because the size of discrete BBNs increases approximately exponentially with the number of discrete states of its nodes. Nojavan et al. (2017) investigated the implications of several mathematical methods for constructing discrete distributions in an unsupervised manner. Using a simple 3-node BBN describing chlorophyll concentrations in Finnish lakes, the authors evaluated the impact on the developed BBNs of the number of intervals and of the choice of the type of discretization methods. Three techniques were investigated, namely in which the data are divided into groups: (1) of equal length; (2) of equal sample size; (3) for which the moments of the discretized distribution match with the moments of the continuous data. They showed that none of the models did uniformly well in all comparison criteria (sum of squared errors, accuracy, area under the receiving operating characteristic curve) for the considered case. They concluded that they cannot justify using one discretization method against others. Using a 4-node BBN from the domain of coastal erosion, Beuzen et al. (2018) extended the tests to other types of discretization methods, namely manual and supervised techniques. They showed, on their specific test case, that supervised methods led to a BBN of the highest average predictive skill, followed by the one with manual discretization. They also outlined the advantages of the different methods, namely that:

- Manual methods allow ensuring physical meaningful BBNs;
- Supervised methods can autonomously and optimally discretize variables and may be preferred when predictive skill is a modelling priority;
- Unsupervised methods are computationally simple and versatile.

Depending on the objective, some specific discretization algorithms have also been developed; for instance, Zwirglmaier and Straub (2016) developed specific methods to deal with rare events in reliability analysis; Neil et al. (2007) proposed a dynamic discretization method to perform inference in hybrid BBNs, i.e. both dealing with continuous and discrete variables.

3.2.4 Combining scarce data and expert judgements

When data are scarce, the parameter learning may be improved by incorporating additional information provided by experts. A popular approach relies on the Maximum a Posteriori (MAP) estimation using Dirichlet priors, which express experts' belief (e.g., Heckerman et al., 1995) about θ in the absence of data. Formally, the Dirichlet distribution for CPT column θ_{ij} is expressed as follows:

$$p(\theta_{ij}) = \frac{1}{Z_{ij}} \prod_{k=1}^{r_i} \theta_{ijk}^{(\alpha_{ijk}+1)-1} \quad (2)$$

with $\sum_k \theta_{ijk} = 1$, $\theta_{ijk} \geq 0$, Z_{ij} is a normalisation term $\int_{-\infty}^{+\infty} \prod_{k=1}^{r_i} \theta_{ijk}^{(\alpha_{ijk}+1)-1} d\theta_{ijk} = 1$, and α_{ijk} is the parameter of the Dirichlet distribution, which can be intuitively interpreted as "how many times the expert believes he/she will observe X_i=k in a sample of α_{ij} instances drawn independently at random from the distribution θ_{ij}" (Zhou et al., 2014). On this basis, MAP relies on the following equation:

$$p(\theta|D) \propto P(D|\theta)P(D) \propto \prod_{ijk} \theta_{ijk}^{(\alpha_{ijk}+N_{ijk})-1} \quad (3)$$

This equation results in the estimate of θ_{ijk} as $\frac{N_{\mathrm{ijk}}+\alpha_{\mathrm{ijk}}-1}{N_{\mathrm{ij}}+\alpha_{\mathrm{ij}}-1}$, which combines information from the data and from the experts' prior guess. In their computer experiments using twelve publicly available BBNs (available at http://www.bnlearn.com/bnrepository/), Zhou et al. (2016a) showed that MAP achieves better performances than conventional MLE, which suffers from the absence of data in several state configurations in situations of limited sample size (typically 50).

Expert-based information can take several forms, and the one that corresponds to qualitative constraints have given rise to several developments. Instead of directly providing the exact value of the entries of binary BBN (denoted $P_{1\text{-}2}$), the expert may feel more conformable in providing an ordering like "$P_1>P_2$", "$P_1\approx P_2$", "$P_1>0.80$", etc. Zhou et al. (2016a) showed that incorporating such expert knowledge about the monotonic influences between nodes (translated into probability constraints) further outperformed MAP and MLE and was also robust to errors in labelling the monotonic influences.

Different methods have been developed to incorporate qualitative constraints, namely:

- Convex Optimization (Niculescu et al., 2006; Zhou et al., 2016a; de Campos and Ji, 2008; Liao and Ji, 2009; Altendorf et al., 2005) is an extension of the MLE by incorporating constraints via penalty functions or by restricting parameter spaces;
- Constrained MAP approach has also been proposed by Yang et al. (2019) to learn BN parameters by incorporating convex constraints;
- Isotonic Regression (Feelders and van der Gaag, 2005; 2006) builds on qualitative information about the influences between the variables of a BBN. The most recent algorithm by Masegosa et al. (2016) also enables the analysist to learn the CPT parameters from incomplete data;
- Qualitative MAP (originally proposed by Chang and Wang (2010) and further improved by Guo et al. (2017)) constructs Dirichlet priors from Monte-Carlo random samples of the constrained parameter space, which are used by the MAP algorithm;
- Multinomial Parameter Learning with Constraints (Zhou et al., 2014; Hospedales et al., 2015) rely on auxiliary BBNs, which are hybrid BBNs, to infer the posterior distribution of BBN parameters.

3.2.5 Discussion

Following a pure statistical data-driven approach for populating the BBN conditional model requires a large amount of statistically significant data to cover all BBN relationships. To compensate the lack of data, a possible option is to complement the analysis with expert-based information. Sect. 3.2.4 shows that a broad range of different tools/methods are available to incorporate expert-based information either in the form of qualitative influences or constraints, namely constraints that should be almost linear and convex (i.e. concave constraints like $P_1 \neq 0.5$ cannot be accounted for). The improvement of the learning accuracy of the parameters in BBNs from a small data set has been shown using each of the described methods compared to conventional methods; for instance Guo et al. (2017) compared MLE, constrained MLE, maximum entropy and constrained maximum entropy estimator, MAP and their qualitatively MAP estimator. Yang et al. (2019) showed the higher performance of their constrained MAP estimator compared to conventional parameter learning algorithms, MLE and MAP, and to constrained maximum likelihood algorithm. Yet, to the author's best knowledge, no extensive benchmark exercise covering all the afore-mentioned estimators (as well as their pros and cons) is available yet; practical recommendations on how to implement them and their limitations is currently lacking in the literature.

Among the possible limitations, the problem of under-fitting related to the use of prior distributions (that are common ingredients of most of the methods of Sect. 2.4) is seldom tackled. As described by Gao et al. (2019), imposing certain a priori knowledge on the CPT parameters might decrease the likelihood of the parameters, hence a reduction of the fitness between parameters and data. Azzimonti et al. (2019) proposed a hierarchical procedure to improve the widely-used approach based on Dirichlet priors. Gao et al. (2019) proposed a Minimax Fitness algorithm combined with an improved constrained maximum entropy method to overcome this problem. They also concluded that there is a need for further investigation to develop learning methods that does not require specification of prior strength.

3.3 Learning from experts

In many situations, the primary source of information for learning the CPTs is not based on data, but on inputs from expert domain. For instance, for rare-event situations like reliability analysis, inputs from expert domain stem from questionnaires, interviews and panel discussions. Sect. 3.3 focuses on the process of deriving information from experts that is named “elicitation”. The issues and methods related to this task were analysed by review articles in different domains of application, namely shipping accidents by Zhang and Thai (2016), human reliability by Mkrtchyan et al. (2015) and more broadly regarding dependence in probabilistic modelling by Werner et al. (2017). The objective is to focus the elicitation on specific pieces of information to efficiently populate the CPTs by ensuring quality and consistency of the elicited result and minimizing the workload on the experts owing to the large number of CPT entries. Elicitation for CPT generally relies on three (possibly combined) main approaches through: (1) the assessment of probabilities directly from an (or a panel of) expert (Sect. 3.3.1); (2) assumptions on the causal structure either by simplifying the network structure or by simplifying the causal dependence (Sect. 3.3.2); (3) filling-up methods (Sect. 3.3.3).

3.3.1 Direct elicitation

In a direct approach, experts are asked to give quantitative numbers (like frequencies or confidence intervals) using methods like probability wheel, probability scale and gambling analogy. Extensive discussions on the different types of biases are provided by Renooij (2001), and more specifically in the domain of ecology by Kuhnert et al. (2010). Overall, methods which map qualitative statements to numerical values like the probability scale (see an example in Fig. 2(A)) is preferred for its simplicity, which improves the consistency (as underlined by Wiegmann (2005), and as reported by Zhang and Thai (2016) for marine safety). Probability wheel is criticized for not being appropriate for the elicitation of small or large probabilities, and the gambling analogy is criticized for being too time-consuming.

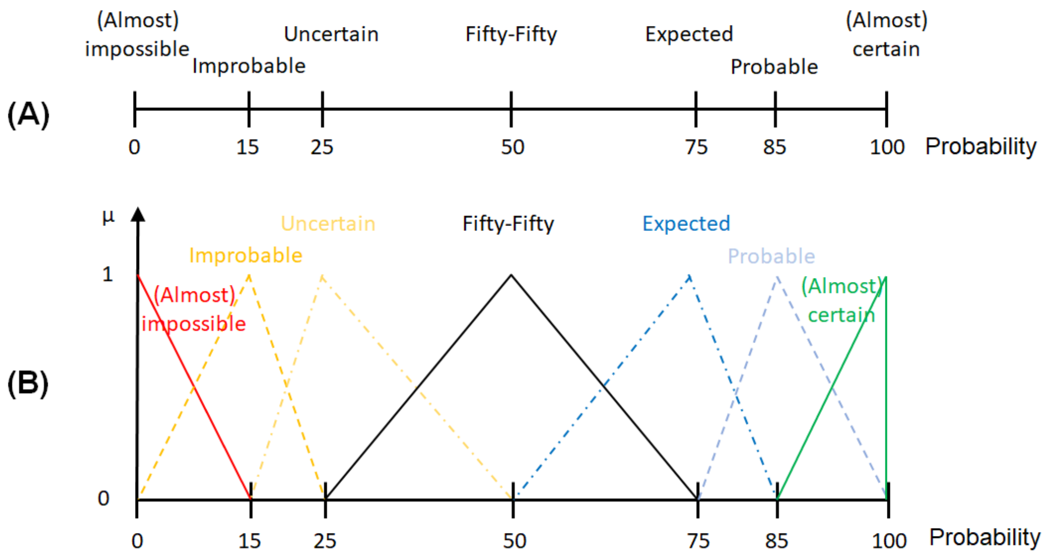

Figure 2: (A) Example of probability scale used to assist expert elicitation of CPTs (adapted from Knochenhauer et al. (2013)); (B) Translation of the probabilities qualified in (A) into Fuzzy sets (μ is the degree of membership).

As an alternative, experts are preferably asked to give qualitative statements (like categorical or relative measure). To support this indirect approach, tools from the domain of multicriteria decision-making have been proposed. For instance, Chin et al. (2009) adapted the Analytical Hierarchy Process method for the task of probability elicitation and semi-automatic generation of the parameters of CPTs. The basic idea is to elicit paired comparisons about the relative likelihood of the possible events using predefined scores (equally possible, etc.) instead of directly asking the probability values. Yet, this procedure is at the expense of an increase in the number of comparisons as the number of conditional probabilities increases.

An alternative option proposes to directly process natural linguistic terms by mathematically modelling them using for instance a Fuzzy set (Zadeh, 1975). Let us consider the concept of membership function, which defines how each element x of the input space X (also named "universe of discourse") is mapped to a degree of membership (denoted μ). Under the classical theory of Boolean logic, the membership function of a set A is simply defined as a binary function that takes the value $\mu(x)$ =1 if the element belongs to A and the value $\mu(x)$=0, otherwise. The Fuzzy set theory of Zadeh (1965) introduces the concept of a set without a crisp (i.e. clearly defined) boundary. Such a set can contain elements with only a gradual (partial) degree of membership (μ is scaled between 0 and 1). The translation of the probability scale of Fig. 2(A) into Fuzzy sets is provided in Fig. 2(B). Some successful applications cover fault detection (D'Angelo et al., 2014), performance analysis of devices (Penz et al., 2012), safety risk analysis (Zhang et al., 2015), human reliability analysis (Li et al., 2012), and offshore risk (Ren et al., 2009). Two viewpoints exist in the literature on Fuzzy BBNs. Fuzziness can be incorporated in the variables (nodes) or on the probabilities. For instance, Ren et al. (2009) carried out studies using fuzzy probability calculations in BBNs (as illustrated in Fig. 2(B)). Conversely, Tang and Liu (2007) used fuzzy events (i.e. Fuzzy node states) in BBNs for a machinery fault diagnosis problem. İçen and Ersel (2019) incorporated both aspects with application in medicine. A more extensive analysis of methods based on Fuzzy sets are investigated in Sect. 5.

3.3.2 Making assumptions on the causal structure

To reduce the elicitation burden, the number of CPT entries to be elicited should be kept "reasonable". This can be performed by making assumptions regarding the causal structure. One option is by simplifying the structure through the introduction of "divorcing" nodes (Henderson et al., 2009). This involves aggregating a few of the nodes by adding a new node that summarizes them provided that the aggregations are logical and no interactions are lost in the procedure. Although this process adds nodes to the network, it reduces the combined size of CPTs in the network (Cain, 2001). Yet, divorcing might dilute the sensitivity of the final node(s) to the input nodes and might increase the uncertainty propagated through the network as underlined by Cain (2001).

A popular alternative aims at making some simplifications regarding the causal dependence based on the logical Noisy-OR gate (Pearl, 1988). In their typical implementation, Noisy-OR gates focus on binary BBN nodes and assume that the influence of the considered factor is independent from the presence of the other factors. This means that the probability of the outcome is the product of the probabilities of the outcome in presence of one factor at a time, with all other factors being absent. Formally, let us consider a binary variable Y with two states {False, True} and n binary parent variables $X_{i=1,..,n}$.

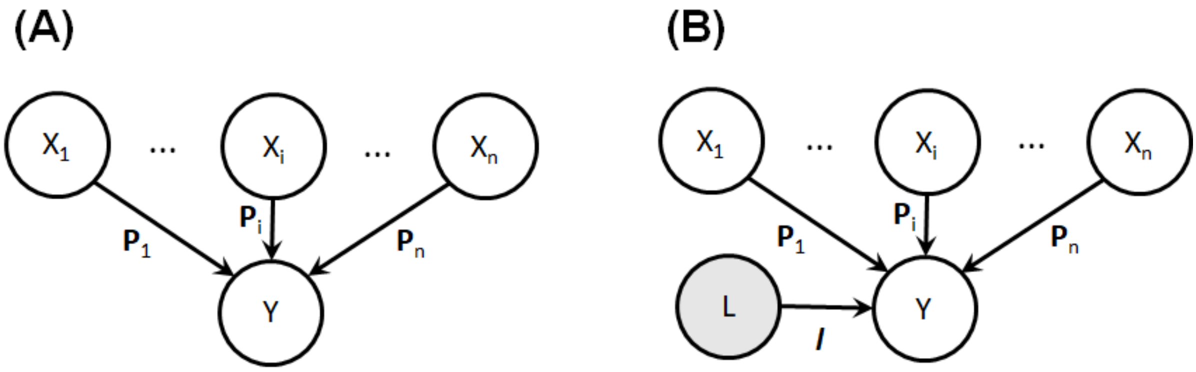

Figure 3: (A) Schematic representation of the Noisy-OR gate with Pi=1,..,n the link probabilities; (B) Schematic representation of the leaky Noisy-OR gate.

The main principle of the Noisy-OR model is to define probabilities P_i (termed as link probability, Fig. 3(A)), which are defined as the probability that Y is False given that X_i is False and X_j is True for $i \neq j$. A Noisy-OR model is thus a disjunction "noisy" version of X_i (Pearl, 1988). This means that the distribution of Y conditional on X_1; X_2;…; X_n is $P(Y = F|X_1; \dots; X_n) = 1 - \prod_{i:X_i \in \boldsymbol{X}_T}(1 - P_i)$ where $\boldsymbol{X}_T$ is the set of parent nodes whose states are True. The Noisy-OR model enables the analysist to specify fewer CPT parameters; the number of independent parameters being here reduced from 2^n to 2n. The extension of Noisy-OR gate to multi-valued variables is the Noisy-MAX gate model (Diez, 1993; Henrion, 1989). If the parent node X_i has n_{X_i} states, then the total number of parameters that have to be elicited using leaky Noisy-MAX gate is $N = \sum_{i=1}^{n}\left(n_{X_i} - 1\right)(n_Y - 1) + 1$ to be compared to the total number without Noisy-MAX gate, namely $N = (n_Y - 1).\prod_{i=1}^{n} n_{X_i}$.

Different empirical studies have been conducted to investigate the performance of the leaky Noisy-OR approach. Several authors (Oniśko et al., 2001; Anand and Downs, 2008; Bolt et al., 2010; among others) showed how this approach helped reducing the burden of elicitation in practical real-life applications without impacting too much the performance of the network. Besides, Zagorecki and Druzdzel (2012) explored to which extent the pattern of causal interaction induced by Noisy-OR(MAX) gates are common in real cases. Using three existing BBNs, they showed that the Noisy-MAX gate provides a good fit for as many as 50% of CPTs in two of these networks.

The Noisy-OR structure is based however on a strong assumption, i.e. that the node of interest is in the state False (considering the above illustrative case) with a probability equal to 1 if all its parent variables are in the state False. Yet, in many cases, it is often difficult to capture all the causes of the node of interest (e.g. for reliability purpose, it means to define all the failure modes of a component). To deal with this problem, Henrion (1989) proposed an extension called "leaky Noisy-OR" gate that includes a background probability that represents the influence of non-modelled causes as schematically depicted in Fig. 3(B). Zagorecki and Druzdzel (2004) proposed to elicit leaky and non-leaky Noisy-OR parameters as alternatives to conditional probabilities using statements like "What is the probability that Y is present when X_1 is present and all other causes of Y (including those not modelled explicitly) are absent?". They showed that the leaky Noisy-OR parameter was assessed as the most accurate (in terms of Euclidean distance to empirical distribution).

The leaky Noisy-OR method was further extended by relaxing the necessity to define a crisp precise leaky probability value, i.e. by introducing uncertainty on this parameter. This type of uncertainty has been addressed within different uncertainty treatment settings (which are introduced in more details in Sect. 3.4). Antonucci (2011) developed an imprecise leaky Noisy-OR gate model with uncertainty on the link probabilities modelled by intervals within the formalism of credal networks (see Sect. 3.4.2). Alternatively, Fallet-Fidry et al. (2012)

(further extended by Zhou et al. (2016b)) proposed an imprecise extensions of the Noisy-OR within the formalism of evidential networks (see Sect. 3.4.3). Finally, Dubois et al. (2017) developed a version of noisy logical gates within the theory of possibility (Dubois and Prade, 1988) using possibilistic causal networks (as presented by Benferhat et al. (2002)) with illustration on an example taken from human geography.

3.3.3 Filling-up methods

Alternative methods to Noisy-OR(MAX) gate are based on filling-up techniques. These methods are typically based on extracting information on the factor effects from known relationships (named anchor conditional probability distributions, denoted CPD) and extrapolating to the whole CPTs. Considering two BBNs (of respectively 3 and 4 nodes) for a human reliability problem, Mkrtchyan et al. (2015) tested five popular methods for CPT derivation considering nodes with multiple states, namely:

- Method 1: the functional interpolation method (Podofillini et al., 2014) approximate CPDs elicited at the anchor positions by functions described by parameters (e.g., Normal functions); the parameters of the missing CPDs are then obtained by interpolating those corresponding to the anchor ones;
- Method 2: the Elicitation BBN method (Wisse et al., 2008) is based on piecewise linear functions interpolating among the elicited CPDs, and on state influencing factors and importance weights;
- Method 3: The Cain calculator (Cain, 2001) uses interpolation factors derived from CPDs to populate the missing relationships in CPTs;
- Method 4: The method presented by Røed et al. (2009) is also based on functional relationships between influencing factors and outcome nodes; the parameters of the function (exponential) are then determined based on the elicitation of selected CPDs;
- Method 5: the ranked node method by Fenton et al. (2007) (further improved by Laitila and Virtanen, 2016) is not based on interpolation of known CPDs. In this approach, all the nodes are defined on the interval [0–1]. For instance, let us consider a node with 5 states, namely ‘‘very low”, ‘‘low”, ‘‘average”, ‘‘high”, and ‘‘very high”; each of the state is assigned to an interval width of 0.2; for instance, the value ‘‘low” is assigned to the interval [0.2–0.4]. To generate CPTs, the experts are asked to provide the weight parameters and to choose one algorithm (the mean average, the Minimum, the Maximum and the MixMinMax). Using this method, if there are m ranked nodes and each node has n states, the expert will only need m+1 parameter values, while it requires $n \times m+1$ values for full elicitation.

Mkrtchyan et al. (2015) showed that:

- All methods allow representing the different importance of the various influencing factors;
- The representation of the interactions (combined effects of multiple factors) is problematic for methods eliciting information on the influence of factors taken one at a time (methods 2-4);
- Functional representation of the CPTs (methods 1, 5 and 4) can be traced more easily, because they allow an explicit representation of uncertainty in the factor relationships;
- But methods 4 and 5 have difficulties in representing the different degrees of uncertainty in the relationships;
- The method allowing the largest modelling flexibility is method 1 with respect to strong factor influences (single and multi-factor) and proper uncertainty characterization, but becomes too costly for large BBNs.

3.3.4 Discussion

The conclusions drawn by Mkrtchyan et al. (2015) serve as valuable recommendations regarding the use and applicability of the five most popular filling-up methods for reducing the expert burden of CPT elicitation. Despite the practical usefulness of this comparative exercise, it should be noted that they primarily focused, by construction, on the modelling aspects important for their application domain (here human reliability analysis), namely the representation of strong factor influences and interactions, and the characterization of different degrees of uncertainty in the relationships. Broader exercises are needed to cover a larger spectrum of methods (i.e. filling-up methods should be completed by Noisy- OR/MAX models, direct elicitation among others), of contexts (different network sizes, binary versus multivalued nodes, etc.), as well as of domains of application.

Despite the clear advantages of these methods for BBN engineering, they cannot be applied uncritically, because the probability values can only be considered approximations of the true probabilities and whatever the considered methods, they are all based on simplifications that may hamper the BBN performance. Initiatives like the one by Woudenberg and van der Gaag (2015) for the Noisy-OR model should be intensified. They identified the conditions under which ill-considered use of this method can result in large impact on output probabilities; in particular, when the yet-unobserved cause variables in the mechanism have relatively skewed probability distributions and/or the obtained parameter probabilities have small values. For this purpose, sensitivity methods as described in Sect. 5 can play an important role. Fenton et al. (2019) also dealt with the limitations of leaky Noisy-OR model for backward inference. When the binary node of interest Y of the example in Fig. 3 is observed to be in the state False, the normal “explaining away” behaviour fails, which means that after observing the state of any parent the remaining parents become independent, and the results may not result in what BBN practitioners expected. Fenton et al. (2019) described a simple extension of the model that requires the elicitation of only one extra parameter that can solve this problem for a large spectrum of cases in practice.

3.4 Propagating the uncertainties

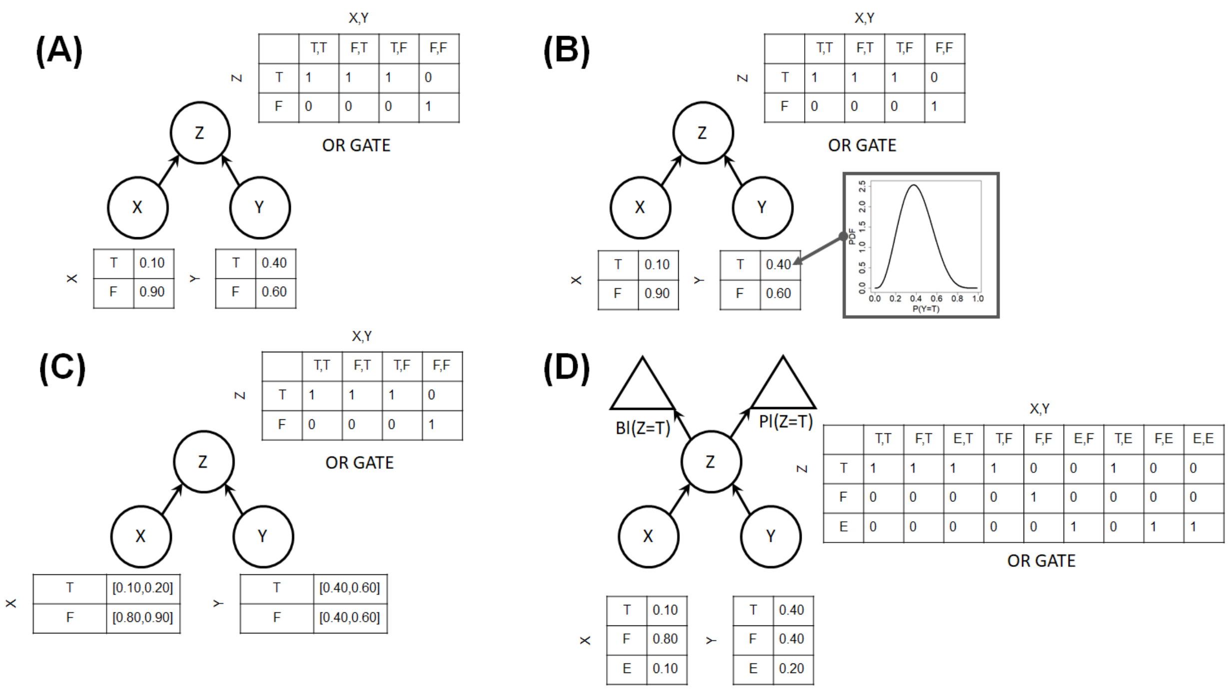

Figure 4: (A) Example of an OR gate model translated into a BBN with two binary parent nodes X and Y (with states corresponding to T=True or F=False). The truth table related to the OR gate corresponds to the table next to the child node Z. Illustration of a probability-based approach where uncertainties on CPT entries are represented by: (B) Beta probability distributions (with an example here for node Y); (C) Interval-valued probabilities (credal network approach); (D) Mass probability tables; here the truth table includes the epistemic state E={T,F}; Two nodes were added to the network to calculate the belief and plausibility functions (see Sect. 3.4.3 for more details).

Whatever the methods used to populate the CPTs, residual uncertainties may still prevail. This residual uncertainty should be reflected in BBN-based results. This means that the uncertainty on CPT entries should be propagated in order to evaluate their consequences on the BBN results. The propagation can either rely on probabilities (Sect. 4.1), or alternative mathematical representation tools like intervals (Sect. 4.2) or a generalization of a probability distribution (Sect. 4.3), i.e. within the theory of belief functions as introduced by Shafer (1976) and Dempster (1967). Fig. 4 summarizes the main principles of the different approaches by using a simple OR-gate model.

3.4.1 Methods using probabilities

The problem of uncertainty propagation for BBN has originally been addressed using probabilities. This approach assumes that the uncertainty on CPT entries follows a Beta probability distribution (or for more generic cases, a Dirichlet probability distribution), as schematically depicted in Fig. 4(B). Kleiter (1996) originally described a Monte-Carlo-based random simulation procedure to carry out the approximation of the spread of the probability distribution for the considered query. The method requires, however, a large number of random samples to accurately characterize the true variance. Van Allen et al. (2008) proposed an improved method by avoiding Monte Carlo sampling through the combination of bucket elimination (Dechter, 1998) with the "delta rule" that linearizes the relationship between the query probabilities and the corresponding Dirichlet conditional probabilities connecting the query variable to its parents and children. They further proved that the Beta approximation (for binary BBNs) is asymptotically valid. The conditions of the exact Beta distribution has extensively been investigated by Hooper (2008). This problem has further been formalized within the setting of subjective logic (Jøsang, 2001; 2016) as proposed by Kaplan and Ivasnoska (2018), who developed an efficient belief propagation for inference in a binary Bayesian network with a singly-connected graph. To introduce any type of probability distribution on CPTs, Fenton (2018) proposed to extend the BBN with continuous nodes corresponding to the uncertain prior probability distributions, but at the expense of a potentially large increase of computational time cost when the number of nodes and of CPTs increases.

3.4.2 Methods using interval-valued probabilities

Instead of specifying a crisp single value of each CPT entry, the formal setting of credal network, denoted CN (Cozman, 2000; 2005), integrates BBNs with credal sets, i.e. set of probability measures. A CN can be viewed as the representation of a set of BBNs, which share the same graphical structure but are associated to different conditional probability parameters; the interest being to provide a richer representation of uncertainty. In Fig. 4(C), the uncertainty in the CPTs are presented by intervals.

Formally, given a variable X, we denote by Ω, the possibility space of X, x a generic element of Ω, $P(X)$ the probability mass function for X and $P(x)$ the probability of x. The credal set over X is $K(X)$, which corresponds to a closed convex set of probability mass functions over X. For any $x \in \Omega$, the lower probability for x according to the credal set $K(X)$ is $\underline{P}(x) = min_{P(X) \epsilon K(X)} P(x)$. Similar expression can be given for the upper probability. Within Walley's theory of imprecise probabilities (Walley, 1991), credal sets can then be represented as polytopes, where each inner point has a valid probability mass, and can be obtained by computing the convex hull of a finite number of probabilities, called vertices (Cozman, 2000). A credal set for a random variable X_i is labelled $K(X_i)$, while the set comprising its extreme points is denoted by $\mathrm{ext}[K(X_i)]$.

A credal network CN (Cozman, 2000) over a set of random variables is thus a DAG where dependencies among variables are defined by a set of conditional credal sets as $K(X_i|pa(X_i))$. By analogy with BBN, it is possible to define a joint credal set as follows:

$$K(X) = CH(P(X): P(x) = \prod_{i=1}^{n} P(x_i|pa(X_i)) \quad (4)$$

where $P(X_i|pa(X_i) \in ext[K(X_i|pa(X_i)]$, CH is the convex hull operator, applied to the probabilities computed for the combination of all the vertices of all the conditional credal sets.

In this setting, the task of inference aims at computing the probability bounds of the largest extension that satisfies the Markov condition (i.e., independence of each node of its non-descendant non-parents given its parents) under the assumption of strong independence (Cozman, 2000). This results in the convex hull of the set containing all joint distributions that factorize the overall joint probability of the network, where the conditional distributions $P(X_i|pa(X_i)=\pi_k)$ are selected from the local sets $K(X_i|pa(X_i)=\pi_k)$. This task is a NP-hard (de Campos and Cozman, 2005), for which a number of exact and approximate algorithms have been proposed (Antonucci et al., 2015; Mauá et al. 2012; Ide and Cozman, 2008; Cano et al., 2007), but only exact inference algorithms are suitable to polytree-shape binary networks.

Though CN allows quantifying and integrating the uncertainty on CPTs on the BBN inference results, this increase in expressiveness comes at the expense of higher computational costs. Some real case applications of CN exist in different domains (Table 1), but the number of them remain limited (with comparison to BBN), despite the availability of some open-source solutions like OpenCossan (Tolo et al., 2018), the linear programming algorithm[2] of Antonucci et al. (2015), the GL2U-II algorithm[3] of Antonucci et al. (2010).

Table 1: Case studies of evidential networks (EN) and Credal Network (CN)

	Domain of application	**Method**	**Reference**
1	Hazard assessment of debris flows	CN	Antonucci et al. (2007)
2	Military identification	CN	Antonucci et al. (2009)
3	Reliability analysis of a fire-detector system	EN	Simon et al. (2008; 2009; 2017)
4	Threat assessment	EN	Benavoli et al. (2009)
5	Convoy detection	EN	Pollard et al. (2010)
6	Reliability analysis of oil filter plug linked to aero engines	EN	Yang et al. (2012)
7	Railway dysfunction	EN	Aguirre et al. (2013)
8	Food processing	Dynamic CN	Baudrit et al. (2016)
9	Cyber attack analysis	EN	Friedberg et al. (2017)
10	Vulnerability analysis of Nuclear Power Plant subject to external hazards	CN	Tolo et al. (2017)
11	Reliability analysis of a safety instrumentation system for a pressurized vessel	EN	Zhang et al. (2017)
12	Medical prognostic and diagnostic	EN and Fuzzy sets	Janghorbani and Moradi (2017)
13	Fault diagnosis for railway	BBN vs EN	Verbert et al. (2017)

[2] http://ipg.idsia.ch/software.php?id=135
[3] http://ipg.idsia.ch/software.php?id=142

14	Maritime accidents	CN	Zhang and Thai (2018)
15	Terrorist attack analysis on a chemical storage plant	EN vs CN	Misuri et al. (2018)
16	Landslides	CN	He et al. (2018)
17	Risk assessment of an oscillating water column	CN	Estrada-Lugo et al. (2018)
18	Reliability analysis of a feeding control system	EN	Mi et al. (2018)
19	Human reliability for Nuclear Power Plant safety analysis	EN	Deng and Jiang (2018)
20	Safety assessment of a truss	EN	Khakzad (2019)

3.4.3 Methods based on Dempster-Shafer Theory

An alternative setting for representing imprecision is the theory of belief functions, also called Dempster-Shafer Theory, denoted DST (Shafer, 1976, Dempster, 1967). Let *X* be a variable taking values in the frame of discernment Θ composed of *q* mutually and exhaustive possible state of *X*. For instance, for a binary node, the frame of discernment is Θ={True, False}. Formally, the theory introduces the concept of basic belief assignment (BBA) based on the belief mass function *m*: $2^{\Theta}\rightarrow[0,1]$ and satisfies $\sum_{A\subseteq\Theta} m(A) = 1$, $\text{and}\, m(\emptyset) = 0$ (which assumes that at least one element of Θ is true). Every $A \in 2^{\Theta}$ such that m(*A*)>0 is called a focal element.

In classical probabilities, a probability value can be assigned to the state True or False only. By defining the belief mass function based on the powerset of the frame of discernment 2^{Θ} (which corresponds in the binary example to {∅,True, False, {True,False}}) enables the analysist to allocate a quantity supporting an additional state termed as epistemic state E={True, False}. Due to uncertainty, the analysist may not always be able to determine the amount of masses to attribute to each state, and the variable *X* may then be in both states, True or False. This means that the method allows characterizing uncertainty about the state of a given node.

From a mass function *m*, two measures can be defined (instead of one for the probabilistic case) called the belief (*Bl*) and plausibility (*Pl*) measures. The latter are respectively defined, for any event *A* as follows:

$$Bl(A) = \sum_{E\subseteq A} m(E), \text{ and } Pl(A) = \sum_{E\cap A\neq\emptyset} m(E) \qquad (5)$$

where *Bl* measures how much event *A* is implied by the information (it sums masses that must be redistributed over elements of *A*), *Pl* measures how much event *A* is consistent with the information (it sums masses that could be redistributed over elements of *A*). These two measures can be associated to a (closed convex) set of bound probabilities {*P* |$\forall A \subseteq \Theta$, (A) ≤(A)≤$Pl(A)$}. It is thus possible to associate an interval-valued probability to the event *A*, with minimum and maximum probabilities provided by *Bel* and *Pl*, respectively. This makes the formal link with CN. Conversely, it is also possible to reconstruct BBAs from *Pl* and *Bel* functions using a Möbius Transformation (Smets, 2002).

As an illustration, let us assume consider a binary node for which the expert only knows that the probability of the event {*X*=True} is at least 0.8. The corresponding BBA is *m*({True})=0.8, *m*(*E*={True, False})=0.2, *m*({False})=0. This means that *Bl*({True})= m({True}), and *Pl*({True})= *m*({True})+ *m*(*E*)=0.8+0.2=1.0. This also means that *Bl*({True})=*m*({True})=0.8, and *Pl*({True})=m({True})+ *m*(*E*)=0.8+0.2=1.0. Then 0.8≤P({True})≤1.0.

The evidence theory is the basis of evidential networks (EN), which is a DAG propagating belief masses. One of the first formulation by Xu and Smets (1996) is based on the Dempster's rule for combining and reasoning with the belief masses. Yet, one major limitation is that the inference algorithms in this formulation are less effective that the one for traditional BBNs as underlined for instance by Khakzad (2019). In the domain of system reliability analysis, Simon et al. (2008) proposed an alternative by mapping logical gates (like OR or AND typically used for fault tree analysis), as EN with the hypothesis described by Guth (1991). Despite its similarity with BBN, relations in EN between variables are not probabilities, but belief masses. The truth table of gates are replaced by conditional mass tables for AND and OR gates (see an example in Fig. 4(D)). To compute belief and plausibility measures in EN, specific nodes (as proposed by Simon and Weber 2009) are introduced. Three types of nodes (as represented in Fig. 4(D)) are thus defined (Simon and Bicking, 2017), namely:

- Root nodes to which BBA are assigned, correspond to components;
- Non-root nodes correspond to logical gates that encode its resulting states {True, False, {True,False}} given the states of its parents;
- Evaluation nodes correspond to nodes that aim at providing estimates of the belief and plausibility measures of the system state.

In the formulation by Simon and Weber (2009), the inference computation is based on the Bayes theorem, which is extended to DST by specifying a mass of 1 on one of the focal elements of the frame of discernment for a specific evidence (hard evidence). Non-specific evidence (soft evidence) corresponds to a mass distribution on the focal elements of the frame of discernment. This means that probability updating in such EN can be based on BBN inference algorithms.

Misuri et al. (2018) compared CN and EN with illustration on a terrorist attack analysis on a chemical storage plant. They highlighted that:

- When used for uncertainty propagation, EN and CN give the same results;
- In terms of implementation, EN is simplier to use, because they can be built using existing codes for BBN, whereas CN requires specific codes;
- In terms of interpretation, Misuri et al. (2018) concluded that EN is more intuitive, because experts directly assign some weight to the epistemic state (e.g. E={True,False} for a binary node), whereas they have to specify interval-valued probabilities for CN, which can become tricky for multivalued nodes.

Khakzad (2019) further filled the gap between CN and EN by proposing some heuristic rules to determine prior belief masses based on imprecise probabilities. They further modified Simon and co-authors' EN formulation to both improve the propagation and updating of the belief masses using BBNs. In order to deal with linguistic variables for the network node' states, the EN method can be combined either with Fuzzy sets (Zadeh, 1975) as applied by Janghorbani and Moradi (2017) for medical prognostic, or with a Naive Bayes classifier model as applied by Zhang et al. (2017) for safety analysis for nuclear power plant.

3.4.4 Discussion

Different settings are available to help the BBN analysist to deal with the problem of uncertainty propagation. A natural question is the justification for using approaches that are alternative to classical probabilities. A first argument often highlights the epistemic nature of the CPT uncertainties. Contrary to aleatory uncertainty (also referred to as randomness), which represents the variability of the physical environment or engineered system under study, epistemic uncertainty mainly stems from the incomplete/imprecise nature of available information (e.g. Hoffman and Hammonds, 1994). While tools from the probabilistic setting can appropriately handle aleatory uncertainties, it is the second type, which raises several problems in practice. In our situation, probability distribution cannot be inferred from data/observations, and should therefore be assumed; the procedure described in Sect. 4.1 is mainly based on the assumption that the uncertainties on the CPTs are described by a Beta (or for more generic cases, a Dirichlet probability distribution) probability distribution. Yet, this assumption may influence the final results of the BBN-based analysis (see. e.g., Ditlevsen, 1994 for an extensive discussion in reliability analysis); Relying only on probabilities masks this problem and might induce an appearance of more refined knowledge with respect to the existing uncertainty than is really present (Klir, 1989; 1994). Sect. 4.2 and 4.3 describe alternative non-probabilistic frameworks to represent uncertainty in situations characterized by limited available pieces of information, which are mainly restricted to expert judgements. Both approaches allow improving the expressiveness with respect to uncertainty representation (as shown by the few tens of application studies using these techniques, see Table 1), in particular by enabling the BBN analysist to translate his/her uncertainty on the node states or his/her imprecision on the CPT parameters by avoiding the need for specifying a probability model.

Yet, extra-probabilistic approaches (whatever the considered methods, CN, EN or networks combined with linguistic variables or based on alternative uncertainty theories like possibility theory, see Dubois et al. (2017)) might come at the expense of higher level of sophistication and of complexity of the inference algorithms (and potentially higher computational costs). The danger is to add more confusion than insights as discussed by Aven and Zio (2011) with the viewpoint of decision making for risk management. The question of selecting the most appropriate approaches for representing and characterizing the risk and uncertainties (in particular with application on BBNs) still remains open (see e.g., an extensive discussion by Flage et al. 2014).

3.5 Characterizing the uncertainties

Methods presented in Sect.3.4 allow evaluating the impacts of CPT uncertainties on the BBN results. But, this tells nothing about the respective contribution of the different CPT entries on the total uncertainty, i.e. the influence of the different uncertainties. This is the purpose of sensitivity analysis (SA), which can be used, in the construction phase of the BBN model, to study how the output of a model varies with variation of the CPT parameters. Subsequently, the results from SA can be used as a basis for parameter tuning, as well as for studying the robustness of the model output to changes in the parameters (Coupé and van der Gaag, 2002; Laskey et al., 1995). This section has strong links with task 3.2.2 of NARSIS WP3.

3.5.1 Description of the methods

For discrete BBNs, a widespread SA method relies on the use of sensitivity functions (Coupé and van der Gaag, 2002; Castillo et al., 1997), which describe how the considered output probability varies as one CPT entry value is changed. An example of application in the domain of marine safety is provided by Hänninen and Kujala (2012). Formally, consider the conditional probability P(Z=k|e), where e denotes the available evidence, and a CPT entry x=P(X=i|π) where i is a value of a variable X and π is a combination of values for the parents of X. The sensitivity function then corresponds to a quotient of two functions that are linear in x of the following form:

$$f(x) = \frac{c_1 x + c_2}{c_3 x + c_4} \qquad (6)$$

where the constants c_i are built from the values of the network's non-varied parameters. The numerator of Eq. 6 expresses the joint probability P(Z=k|e) as a function of x, and its denominator describes P(e) in terms of x. Using the example described in Fig. 1, we focus on the probability of having brain tumor given absence of coma but increased level of serum calcium, i.e. $P(B = T|C = F, ISC = T)$. Van der Gaag et al. (2013) estimated the sensitivity of this probability of interest to the probability of having coma given absence of brain tumor but increased level of serum calcium, $x = P(C = T|B = F, ISC = T)$. The sensitivity function was established as $\frac{-0.03}{x-1.03}$ (as depicted in Fig. 5(A)). This type of function shows that the probability of interest steeply increases when *x* exceeds 0.80, i.e. above the original parametrization given by Cooper (1984).

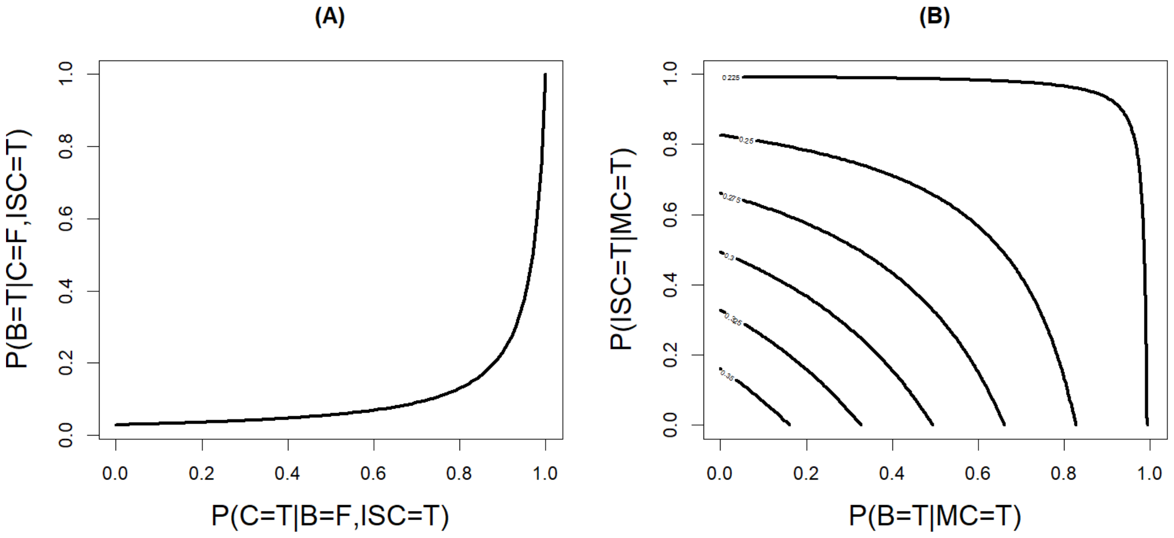

Figure 5: (A) One-way sensitivity function for the BBN described in Fig. 1; (B) Two-way sensitivity function for the BBN described in Fig. 1 considering the probability P(C=T) as the targeted probability.

Two-way sensitivity functions can be expressed in a similar form as a quotient of two bi-linear functions. Consider the sensitivity function of $f(x, y)$ that expresses P(Z=*k*|*e*) as a function of the parameter probabilities $x = P(X = i|\pi_X)$ and $y = P(Y = j|\pi_D)$, where i and j are values of the variables *X* and *Y*, and π_X and $\pi_Y$are combinations of values for the parents of *X* and of *Y*. The function holds as follows:

$$f(x, y) = \frac{c_1 x.y + c_2 x + c_3 y + c_4}{c_5 x.y + c_6 x + c_7 y + c_8} \qquad (7)$$

where the constants c_i are estimated from the values of the network's non-varied parameters. In the tumor BBN example (Fig. 1), we focus on the probability of having a cancer P(*C*=*T*) and its sensitivity to the simultaneous variation of the conditional probabilities *x*=P(*B*=*T*|*MC*=*T*) and *y*=P(*ISC*=*T*|*MC*=*T*), which was established by van der Gaag et al. (2013) as $0.374 + 0.15.x.y - 0.15.x - 0.15.y$ (Fig. 5(B)). This type of function shows that despite the large variation of *x* and *y* (from 0 to 1), the probability of interest varies over a moderate range of values of only ~15%.

A complementary approach for SA involves the study of the Chan–Darwiche (CD) distance (Chan and Darwiche, 2002; 2005), which is a measure for bounding probabilistic belief change. It is complementary in the sense that it gives insight in the effect of parameter changes on the global joint distribution, rather than on a specific (posterior) output probability of interest (as sensitivity functions do). In practices, the CD distance can be used to identify parameter changes, which lead the closer distance between the original and the varied BBN distributions (Chan and Darwiche, 2005). It should however be noted that the choice of the type of distance is rather arbitrary as outlined by Renooij (2014) and other distances like the KL-divergence (Kullback and Leibler, 1951) or the ϕ-divergences (Ali and Silvey, 1966) could also be of interest.

Recent studies have focused on the properties of the SA methods. Renooij (2014) thoroughly investigated the different schemes for varying a probability from a (conditional) distribution, while co-varying the remaining probabilities from the same distribution; the proportional co-variation scheme being the most popular one. Leonelli et al. (2017) further formalized the SA problem for discrete BBNs within the generic setting of multilinear models. They developed a unifying approach to sensitivity methods via the interpolating polynomial representation of discrete statistical models in the context of "BBNs single full CPT analyses", i.e. where one parameter from each CPT of one vertex of a BBN given each configuration of its parents is varied. This approach based on multilinear probabilistic models enabled them to address the problem of multi-way SA (with dimension ≥2). Furthermore, they proved the optimality of proportional covariation by showing that the CD distance is minimized when parameters are proportionally co-varied.

3.5.2 Discussion

Since the nineties, the BBN community has seen the developments of SA methods that are specifically dedicated to their respective needs regarding the BBN use and application. Though simple and efficient to implement, the approach based on sensitivity functions (combined with CD-distance analysis) remains local, because one parameter values are varied, while the other ones are kept constant. Multi-way SA methods have been proposed, but can rapidly become intractable. Interestingly, outside the BBN community, the problem of SA is commonly addressed with alternative tools; variance-based global SA techniques being the most popular one (Iooss and Lemaitre, 2015). Such techniques were adapted by Li and Mahadevan (2018) to bridge the gap between both communities. Their approach has the advantage to be global i.e. in the sense that all CPT parameters' values are changed all together. Besides, this approach can be applicable to any types of BBN (discrete, hybrid or continuous), i.e. it is model-free.

3.6 Summary

In WP3 of NARSIS project, BBNs are developed to perform risk and safety analysis in WP4 and WP5. The current survey has investigated how to deal with uncertainties related to the specification of the main parameters of the BBNs, namely the entries of the Conditional Probability Tables. Three questions were addressed, namely: (1) how to constrain the uncertainties related to CPT derivation; (2) how to integrate these uncertainties in the BBN-based analysis; (3) how to test the robustness of the BBN-based results to these uncertainties. Table 2 provides a summary of the main methods/approaches (together with their advantages and limits) to answer these questions.

Table 2: Summary of the advantages and limits of the main approaches

Question	Approach	Advantages	Limits	Section
1	Learning CPT by combining data and expert prior knowledge via MAP estimation	- It improves the MLE-based fitting when the number of data is limited.	- The representation of expert belief is restricted to the use of Dirichlet priors; - There is a possible problem of "under-fitting" in sparse situations.	Sect. 3.2.4-3.2.5
1	Learning CPT by combining data and qualitative constraints	- The accuracy of the MLE/MAP-based fitting is largely improved when data are scarce; - The experts	- Many new estimators are available, but many lack practical recommendation s;	Sect. 3.2.4-3.2.5

		may feel more conformable in providing ordering than precise CPT values.	- There is a possible problem of "under-fitting" depending on the chosen priors.	
1	Direct elicitation using qualitative statements	- The experts may feel more conformable in providing qualitative statements than quantitative estimates.	- Mathematical modelling of linguistic terms may lead to information loss or increased computation burden.	Sect. 3.3.1
1	Use of "divorcing" nodes	- The number of nodes is decreased through aggregation of nodes.	- Care should be paid to avoid the loss of interactions in the procedure; - It may dilute the sensitivity of the final node(s) to the input nodes; - It might increase the uncertainty propagated through the BBN.	Sect. 3.3.2
1	Simplification of the causal structure using logical gates (e.g. Noisy-OR gate)	- The number of nodes to be elicited is largely decreased (e.g. from from 2^n to 2n for a binary node with n parents).	- The assumptions on the causal relationships might not always be valid in real life applications; - The simplifications may hamper the BBN performance.	Sect. 3.3.2
1	Extracting information on the factor effects from known relationships and extrapolating them	- A large variety of different "filling-up" methods exist to relieve the elicitation burden; - Some feedbacks on real case applications exist (e.g. for human reliability analysis).	- Simplifications are introduced and the derived probabilities can only be considered approximations of the true probabilities.	Sect. 3.3.3
2	Uncertainty	- The degree of confidence in	- The uncertainty representation is	Sect.

	propagation using probabilities	the BBN-based results can be quantified.	restricted to the use of Beta/Dirichlet probability distributions; - It can become computationally intensive.	3.4.2
2	Uncertainty propagation using intervals with credal networks (CN)	- It avoids selecting a probability model to represent the uncertainty; - The experts may feel more comfortable in assigning intervals than probabilities.	- The specification of interval-valued probabilities can become tricky for multivalued nodes; - It needs specific sophisticated inference algorithms and software solutions (with potential high computational costs).	Sect. 3.4.3
2	Uncertainty propagation within the Dempster-Shafer Theory by using evidential networks (EN)	- The expressiveness is improved like for CN; - EN is more intuitive than CN, because experts directly assign some weight to the epistemic state; - It can be implemented with existing BBN softwares.	- The translation of interval-valued probabilities within this setting can become difficult for multivalued nodes; - The inference algorithms for combining joint/disjoint belief masses are not so effective as those based on probability theory.	Sect. 3.4.4
3	Sensitivity Analysis (SA) using sensitivity functions	- The theory is well-established; - It is simple to implement; - The graphical representation is straightforward to interpret.	- It focuses on the influence of one (or multiple) CPT parameters while the other ones are kept constant; - It requires specific co-variations schemes; - Multi-way SA can rapidly	Sect. 3.5.1

			become intractable.	
3	Sensitivity Analysis using Chan–Darwiche distance	- It complements the sensitivity functions by giving insight in the effect of parameter changes on the global joint distribution.	- It presents the same disadvantages than the sensitivity functions; - The choice of the distance can be rather arbitrary.	Sect. 3.5.1

Considering the first question, we described methods for deriving CPT entries from different sources of information (observations, prior knowledge, expert-based information, etc.). Traditional estimators like MLE and MAP (or new ones) were proposed to make the best use of the data available even in scarce situations when completed by qualitative constraints like knowledge about the monotonic influences between nodes.

For rare-event situations as it is the case for scope of NARSIS project, the main source of information relies on inputs from expert domain using different elicitation techniques; the main challenge being the minimization of the workload on the experts owing to the large number of CPT entries while preserving the quality and consistency of the elicited result. Elicitation for CPTs generally relies on three (possibly combined) main approaches, through:

(1) the assessment of probabilities directly from an expert or a panel of experts: this can rely on techniques as described in Sect. 4;

(2) a simplification of the causal structure using the popular easy-to-use Noisy-OR(MAX) model (and its improved versions like the leaky one) whose efficiency has been extensively investigated by Zagorecki and Druzdzel (2012). In particular, its interest for NPP-related human reliability analysis has been shown by Galan et al. (2007);

(3) filling-up methods, which have in particular been thoroughly benchmarked on test cases in the domain of human reliability analysis (Mkrtchyan et al. (2015)). The conclusions (Sect. 3.3.3) are valid outside the domain of human reliability analysis, and can serve as valuable recommendations for NARSIS project.

The second question can be addressed using different approaches, either using probabilities, or imprecise probabilities either using interval-valued probabilities within the setting of credal networks or within the Dempster-Shafer theory within the setting of evidential networks. The main benefit of the latter approach is to allow an increase in expressiveness with respect to uncertainty representation. This isas shown by the few tens of application studies using these techniques (see Table 1), and more particularly in NPP-related analyses as conducted by Tolo et al. (2017) and by Deng and Jiang (2018). We note however that these benefits might come at the expense of higher complexity of the inference algorithms (and higher computational costs).

Finally, the third question is investigated by methods specifically developed for sensitivity analysis of BBN; in particular through the use of one- or multi- way sensitivity functions. The one-way approach has been adopted in WP3 since it has proven to be very efficient in many applications and simple to implement. However, this technique is restricted to the situation, where the other BBN nodes are not perturbed, and the results are dependent on this assumption. Multi-way approaches can overcome this problem but are difficult to interpret and to implement. Further developments are conducted to propose an easy-to-use multi-way method (NARSIS task WP3-Sub-Task 3.2.2).

4 Evaluation of expert-based information

One possible option to constrain the uncertainties related to expert-based information in safety analysis (and in BBN-based analysis, in particular) is to combine the pieces of information provided by a panel of experts i.e. to combine several sources of information. Yet, a necessary preliminary step is to assess the "quality" of these sources of information, i.e. this raises the practical question of the evaluation of expert knowledge. To investigate this question, we aim a comparing two approaches, namely the classical model by Cooke (1991) within the framework of probability theory, and the one within the possibility theory (Dubois and Prade, 1988). In Sect. 4.1, we first introduce the key concepts underlying both approaches, and in Sect. 4.2, we define a comparison exercise that is applied on two expert judgement datasets taken from two domains of interest for NARSIS project, i.e. nuclear safety analysis, and natural hazard assessment. On this basis, we formulate practical recommendations.

4.1 Approaches for expert knowledge evaluation

We first present the principles of expert knowledge evaluation using concepts from in the field of metrology to quantify the quality of a measurement process (Sect. 4.1.1). We then describe how these concepts have been transposed by R. Cooke to propose a formal method of evaluating expert opinions (Cooke, 1991), i.e. to develop the most widely used method named classical model (Sect. 4.1.2). After having discussed the main reasons for using another formalism (Sect. 4.1.3), we present how the concepts of metrology and the method of seed variables can be transposed in the framework of possibility theory (Sect. 4.1.4).

4.1.1 Key concepts in metrology

In metrology, a quantity is defined as the property of a phenomenon, body, or substance, that can be expressed as a number and the quantity intended to be measured is called the "measurand". The objective of a measurement process is to attribute a value as closely as possible of the measurand value. It is also expected that the information from a measurement process permits the assignment of an interval of reasonable values. So that, in metrology a measurement result is a set of quantity values attributed to a measurand together with any other available relevant information. A measurement result generally contains "relevant information" about the set of quantity values such that some may be more representative of the measurand than others. This may be expressed in the form of a probability density function (PDF). Another way to express a measurement result is to return a single measured quantity value and a measurement uncertainty.

Thus, we see two concepts appear to account for the quality of information provided by a measurement process.

- The first one is trueness and refers to the closeness of agreement between the average of an infinite number of replicate measured quantity values and a reference quantity value;
- The second one is precision and refers to closeness of agreement between indications or measured quantity values obtained by replicate measurements on the same or similar objects under specified conditions;
- From these two concepts, a third one is derived and is called accuracy and can be understood as closeness of agreement between measured quantity values that are being attributed to the measurand.

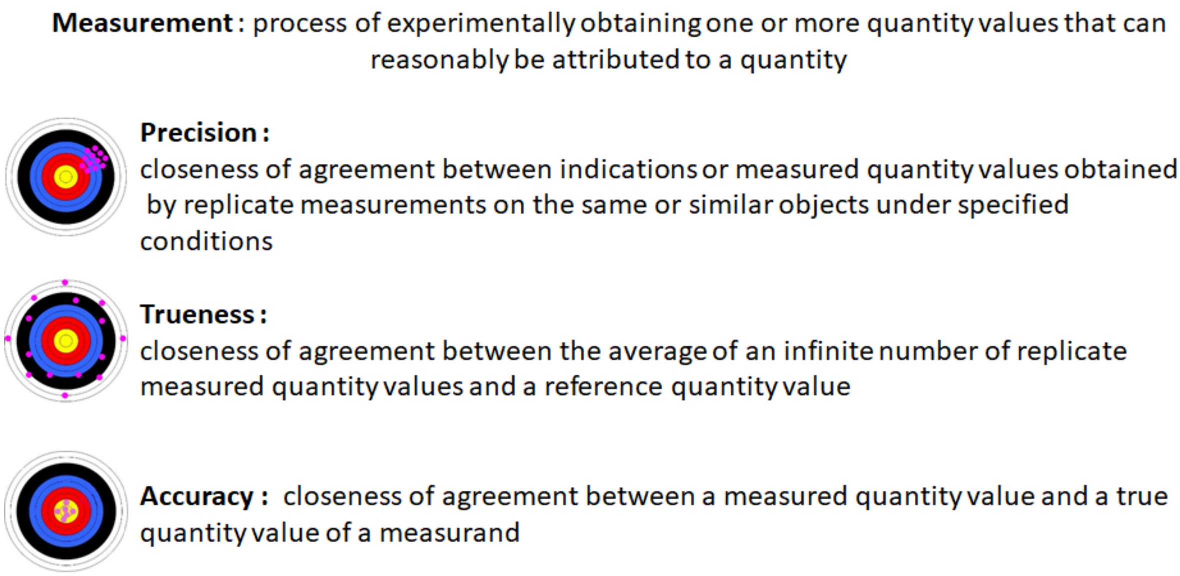

Figure 6: Summary of the concepts in metrology adapted for expert-based information evaluation

To sum up, metrology defines the measurement process as a way to attribute a value to an unknown quantity: the measurand. The quality of the measurement process is derived by replicate measurements and synthetized through trueness, precision and accuracy indicators. See Figure 6 for a summary.

To mimic and transpose the quality indicators defined in metrology to the expert knowledge. The measurand is defined as the unknown quantity for which we ask the expert. As it is expected that repeating the same question to the same expert would lead to the same answer, a possible option is to define “seed variables”. Seed variables are a set of known variables considered similar in terms of expertise as the one for which the expert knowledge is required. The underlying idea is to consider the set of attributed values given by the expert to the seed variables as replicate measurements from which quality indicators can be derived.

4.1.2 Cooke’s formalism

Cooke’s model initially consists of defining a set of variables close to the one for which the opinion of experts is sought. For each variable, the expert is asked to provide confidence intervals expressed by means of percentiles (typically, the 5%, 50% and 95% percentiles).

Let us consider X the unknown variable, P a probability measure on X. The k% percentile, denoted q_k%, is the deterministic value x s.t. P($X \leq x$)=k%. If B+1 percentiles values have been given by the expert (including the lower bound l=q0% and upper bound u=q100%), then the corresponding probability density $\boldsymbol{p}=(p_1,\ldots,p_B)$ is an histogram made of B inter-percentiles (the value of an inter-percentile being the difference between two successive q_k% values). At the end of this process, the information provided by each expert is encoded by an empirical probability distribution (one distribution per seed variable and per expert).

To transpose the concept of trueness and precision, Cooke introduces the concepts of informativeness and calibration. These concepts are a way of providing quantitative measures of the uncertainty of the information given by an expert for each seed variable and of their adequacy with the known values of these variables.

Informativeness

To measure informativeness, the idea is to model the minimum information for each seed variable by a uniform probability distribution whose support contains all the possible values assigned by the experts. Thus, to each seed variable is attached a probability distribution modelling its minimal information (since each expert provides more specific information). In probability theory, a measure of distance between two probability distributions *p*,*q* is given by the relative entropy or KL (Kullback-Leibler) divergence (Kullback and Leibler, 1951) formally defined (discretized version) as follows:

$$\mathrm{KL}(p,q) = \frac{1}{N}\sum_{i=1}^{N} p_i . log(\frac{p_i}{q_i}) \quad (8)$$

The informativeness encoded by a probability distribution is then calculated from its distance to the minimal information distribution, i.e. the uniform distribution: the farthest meaning the most informative. Some examples are provided in Figure 7. From this procedure, as each expert provides a probability distribution for each seed variable, we obtain a quantitative measure of informativeness per expert and per seed variable. To derive a global informativeness score of each expert, the Cooke's proposal is simply to average the informativeness score obtained on the different seed variables.

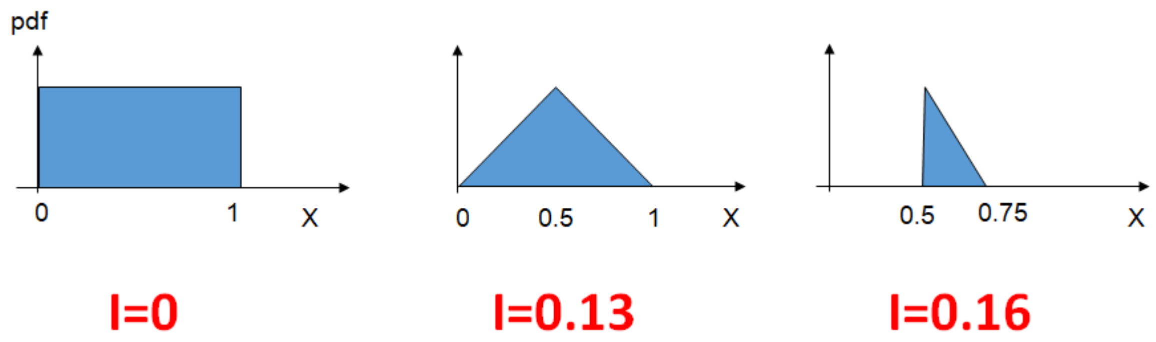

Figure 7: Examples of pdf and corresponding informativeness score (denoted I). The higher I, the more informative.

Calibration

The calibration intends to compare the adequacy between the information provided by the expert and the known values of the seed variables. To quantify a calibration score, Cooke proposes to build and adimensional empirical distribution and to evaluate its distance with the flat distribution between 0 and 1. For example, if we have N seed variables and the expert has given B percentiles (q_1,…, q_B) for the seed variable *S*, one can build an empirical distribution by (p_1,…,p_B, p_{B+1}) where:

- p_j is the ratio of seed variables comprised between q_j and q_{j+1} for for j≠1 and j≠B;
- p_1 (resp. p_{B+1}) is the ratio of seed variables values which are lower (resp. larger) than the percentile q_1 (respectively q_B).

In this way, an expert is perfectly calibrated if the distribution of the seed variables are distributed according to the percentiles, which can be measured using *KL* distance.

Let us take a numerical example to make the calibration procedure clearer. Twenty seed variables are considered and the expert is asked to provide their 5%, 50%, 95% percentiles. Now let us assume that the comparison with the seed variables values and their percentiles gives (1,9,9,1) – situation (a) in Figure 8. This means that the value for 1 seed variable is below its percentile 5%, for 9 seed variables, their values are located between the percentile 5% and the percentile 50%, for 9 other seed variables their values are located between the percentile 50% and the percentile 95% and the last seed variable value is higher than its percentile 95%. In this case, the considered expert is “perfectly calibrated”, because the distribution of the seed variables are distributed according to the percentiles, and the calibration score is here 1. On the contrary, an expert providing (20,0,0,0) – situation (b)in Figure 8, means that, for all the seed variables, she/he is overestimating the possible values, and the calibration score is 0.

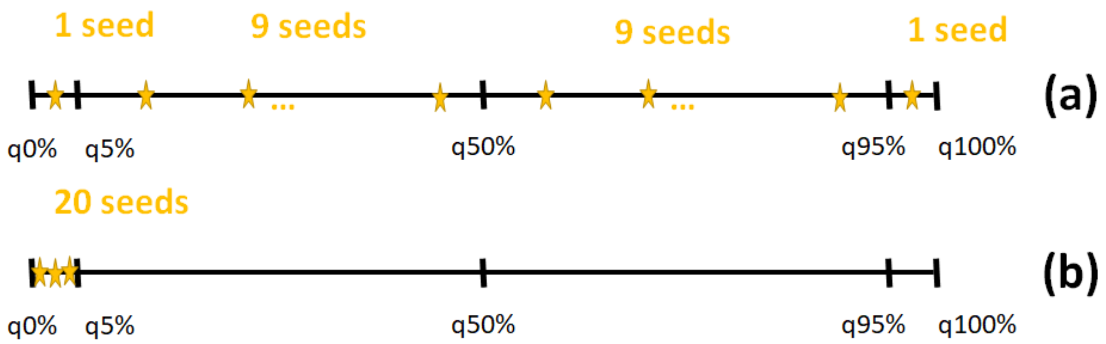

Figure 8: Two examples of repartition of the answers of an expert to 20 seed questions (qk% corresponds to the k% percentile). Situation (a): the expert is perfectly calibrated; situation (b), the expert’s calibration score is zero.

To consider both the informativeness and the calibration, a global score is defined as the product of informativeness and calibration scores. Two main ways of aggregating the expert opinions are usually considered: i) best expert according to the global score; ii) weighed averaging of the probabilities using the global scores of the classical model (see justification by Cooke, 2015).

4.1.3 Why bother with other formalisms?

Cooke’s formalism is a clever way of transposing the concepts used in metrology to evaluate the quality of a measurement. In particular, we think that the introduction of the concepts of informativeness and calibration are two relevant ideas that should be kept in formal elicitation methods. Therefore, the main interest of another formalism mainly resides in its capacity to simply account for the concepts of informativeness and calibration in a scientific community.

First, the meaning of the calibration score appears to be debatable. Let us we consider the afore-described example with 20 seed variables and now consider two experts for whom the comparison with the values leads to the tuples (1,9,9,1) and (0,10,10,1). Do we really want to say that the first one is globally better calibrated, while the second one gives a better uncertainty range for each seed variable?

Second, it appears that the mathematical formulation of a distance between probability distributions (here based on the KL divergence) may not be intuitive to understand, even too complicated; especially when the uncertainty range provided by two experts are different. From this viewpoint, it seems much easier to ‘catch the idea’ of informativeness with fuzzy numbers instead of probability distribution: the calculation of the informativeness score is then reduced to the calculation of an area (see Sect. 4.1.4).

Third, it seems interesting to have different formal methods to quantify the quality of the information, as it is done in metrology, where different measurements devices are usually used.

Finally, the use of another formalism allows to enlarge the panel of fusion operators available for the aggregation of several experts' opinions.

4.1.4 The possibility theory

Probabilities have long been used as a means of assigning a degree of confidence to an assertion. See for instance the original definition by Laplace: "The theory of probabilities is at bottom only common sense reduced to calculation; it makes us appreciate with exactitude what reasonable minds feel by a sort of instinct, often without being able to account for it". Even more explicitly, Finetti's definition in the famous 'Theory of probability' book (subtitled 'Probability does not exist') emphasizes that probability is in fact only an epistemic (and not ontic) measure or a degree of confidence.

From this definition of probabilities, one can then ask whether the axiomatic of Kolmogorov and in particular, the additivity property that is required. Based on this idea, researchers in artificial intelligence have proposed non-additive confidence measures. Possibility is one of the measures of quantification of degrees of belief (Dubois and Prade, 1988). The main difference between measures of possibility and probability is that the possibility of a statement does not imply the impossibility of its opposite.

This makes it easier to model ignorance: I don't know if the covid19 lockdown period will last two weeks or two months is modelled by:

possibility(lockdown=2 weeks) = possibility(lockdown=2 months) = 1

In the case of quantities like the measurand in metrology, it is practical to associate the measures of possibility with fuzzy numbers. Formally, the possibility distribution π can be viewed as determined by the membership function μ of a fuzzy set F. In this vision, $\pi_x(u)=\pi(x=u|F)$ estimates the possibility that the variable x is equal to u, knowing the incomplete state of knowledge "x is F". Then, $\mu(u)$ estimates the degree of compatibility of the precise information $x=u$ with the statement to evaluate "x is F" (Dubois et al., 2000).

The measurand is in this case modelled a fuzzy number, i.e. a convex curve, whose maximum value is equal to 1 and the minimum value to 0. The set of non-zero values is called the support and corresponds to a bounded interval. First, the experts are asked to associate a fuzzy number to each seed variable. This fuzzy number can be defined through a tuple of values playing the role of the quantiles used in the framework of probabilities. For instance, the tuple (lower bound l, ref-, ref+, upper bound u) can be used to model a trapezoidal fuzzy number, for which the membership value is 0 for the values outside [l, u], equal to 1 for values within [ref- , ref+] and linearly interpolated between these two cases. This situation corresponds to the case where the expert affirms that all the values between ref- and ref+ are possible and all the values outside of l,u impossible.

Figure 9 presents a Fuzzy number to represent the knowledge of an expert on the Covid lockdown time duration, with l=1 month, u=3.5 months, ref-=1.5 months and ref+=2 months.

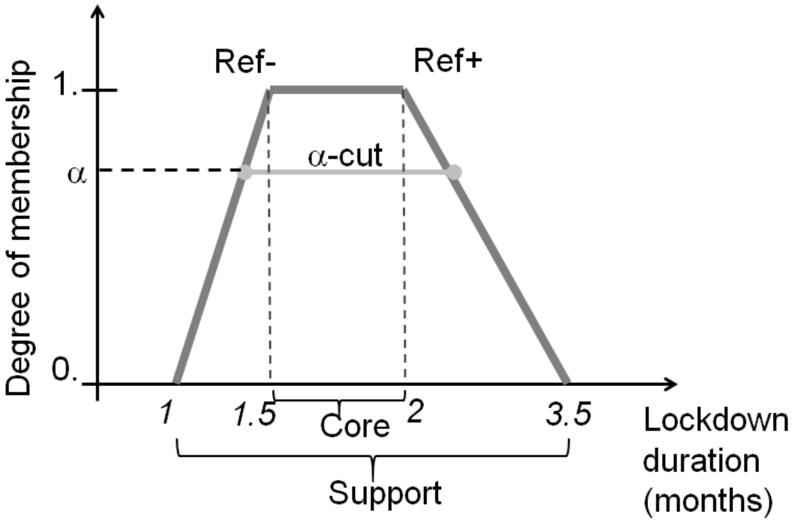

Figure 9: Example of a Fuzzy number to represent the expert-based information on Covid time duration as possibility distirbution

It is possible to adapt the concepts of trueness, precision and accuracy of metrology to the framework of possibilities by using seed variables.

As in the probabilistic method, it is necessary to define a minimal state of information. This is defined as a flat fuzzy number (*l, u*) equal to 1 between min and max and 0 outside. Usually we take for *l* (respectively *u*), the minimum (resp. maximum) possible value of all experts. The informativeness index for a seed variable is then calculated as the complement to 1 of the ratio between the area of the fuzzy number given by the expert and the area of the fuzzy number of minimal information. Figure 10a,b provides two examples, where the blue distribution is the flat fuzzy number (*l, u*) equal, and the triangular ones give the respective information of both experts. In this example, the second expert is less informative than the first one (compare the area in Figure 10b to the one in Figure 10a). The informativeness score of an expert is then the average of informativeness scores obtained for all the seed variables.

For each seed variable, the membership of the corresponding fuzzy number at its calibration value gives the calibration score. Figure 10d,d provides two examples, where the second expert is less calibrated than the first one (compare the degree of membership in Figure 10d to the one in Figure 10c). The calibration score of the considered expert is then the average of all calibration values. Interestingly, the use of fuzzy numbers allows the evaluation of calibration measures to each seed variables contrary to the classical model.

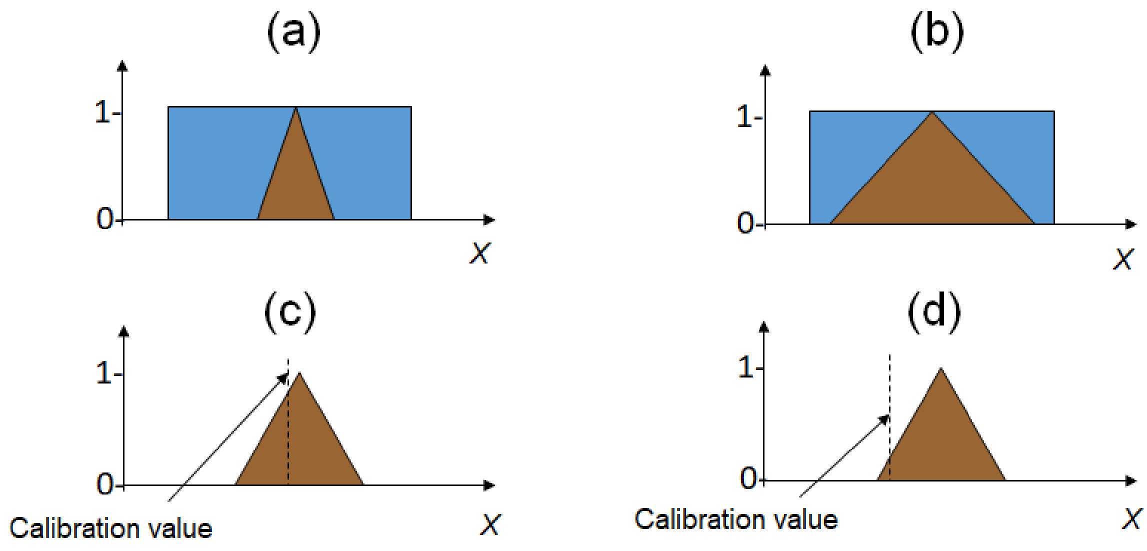

Figure 10: Examples of expert-based information for a given variable X using Fuzzy numbers. (a,b): illustration of how informativeness is measured as the area between the blue and the brown distribution; here expert (b) is less informative than (a). (c,d): illustration of how calibration is measured as the degree of membership of the Fuzzy number at the value of calibration value; here expert (d) is less calibrated than (c).

Finally, the global score is calculated as the product of the informativeness and calibration scores. Similarly to the classical model, two main ways of aggregating the expert opinions can be used: i) best expert according to the global score; ii) weighed averaging of the possibility distributions using the global scores. An example is provided in Figure 11d. The benefit of using the possibility is also to be able to use alternative aggregation techniques (Figure 11b,c). This has extensively been studied by Baccou and Chojnacki (2014) in the domain of nuclear safety.

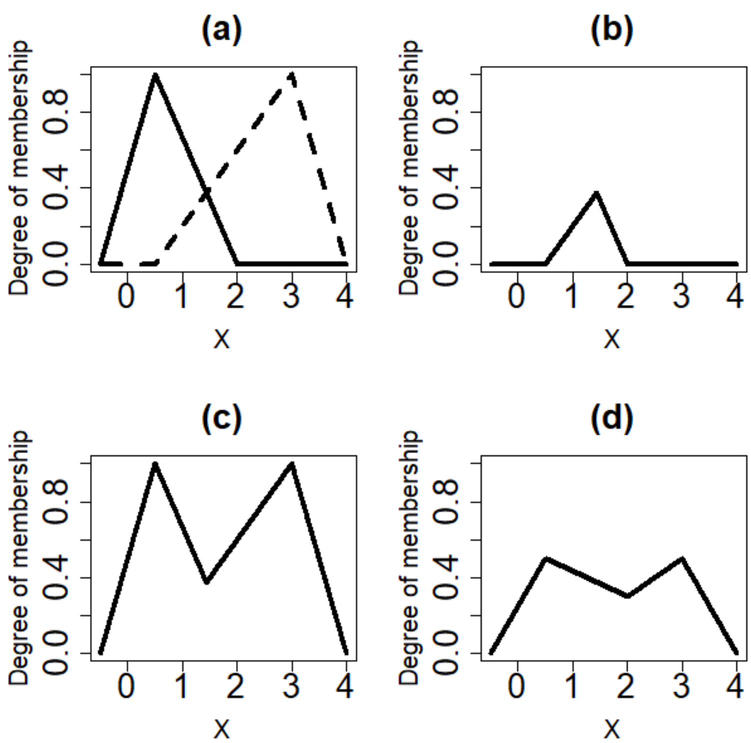

Figure 11: Examples of aggregating two Fuzzy numbers (straight and dashed black lines in (a)) with different methods: (b) union operator (max); (c) intersection operator (min); (d) averaging operator.

4.2 Comparison of both approaches

Based on the concepts introduced in Sect. 4.1, we compare here both approaches for expert-based information evaluation. In Sect. 4.2.1, we first define the setting for conducting the comparison exercise. We then apply it on two real cases (Sect. 4.2.2 and 4.2.3). On this basis, we formulate practical recommendations (Sect. 4.2.4). All computations were performed using in-house scripts with the open source R language (R Core Team (2019)) and environment for statistical computing. An interactive web app is available here: https://github.com/rohmerj/ExpertScoring.

4.2.1 Procedure description

In this section, we aim at comparing both approaches for expert knowledge evaluation. A two-stage procedure is proposed following the terminology of the machine learning domain:

1) Given a set of seed questions, the first stage ('*training set analysis*') aims at identifying and analysing the differences in the ranking of the expert with respect to the difference in the scoring measures, i.e. statistical accuracy (calibration) following the "classical" approach; calibration following the possibility interpretation, and both informativeness scores;

2) The second stage ('*test set analysis'*) takes the perspective of prediction and aims at assessing whether the scores computed on one set of seed questions perform well on another set. This analysis aims at addressing the question: is it better to aggregate experts' judgements following the classical or the possibilistic approach to make prediction?

The second stage is addressed using a cross-validation procedure as implemented for instance by Colson and Cooke (2017). We opt for a leave-one-out cross validation procedure, which consists in:

i) removing in turn each seed question;
ii) aggregating the experts information for the removed seed question;
iii) making the prediction and comparing the results to the true answer.

Four different ways of aggregating the expert opinions are considered: i) best expert according to the classical model (DM_{bestC}); ii) best expert according to the possibilistic approach (DM_{bestP}); iii) weighed averaging of the probabilities using the scores of the classical model (DM_{avgC}); iv) weighed averaging of the possibility distributions (DM_{avgP}).

The difficulty however is to compare the performance of approaches that address the elicitation task with different interpretations and tools for processing the expert knowledge (knowledge representation, aggregation operator). To find a common setting of comparison, we propose to evaluate the performance by taking a pragmatic approach, i.e. the viewpoint of the decision-maker by following the same spirit as the performance measures of the IDEA protocol (e.g., Hemming et al., 2018). From this viewpoint, an expert is expected to show:

- *Accuracy*. It is intuitively understood as the degree to which predictions correspond with observed experimental results. Using range-coded responses, it is calculated as the average log-ratio error ALRE (McBride et al., 2012: Eq. 5) for the r-th expert as follows:

$$\mathrm{ALRE} = \frac{1}{N}\sum_{i=1}^{N} \left|\log_{10}\left(\frac{x_{i,r}+1}{b_{i,r}+1}\right)\right| \quad (9)$$

where N is the number of seed questions, x is the true answer (scaled between 0 and 1 using the lower and upper bound (l,u) of the considered variable), b is the best estimate provided by the expert (assumed to be the median in the classical model and the value with maximum degree of possibility in the possibilistic approach);

- *Informativeness* is understood as the precision with respect to the uncertainty bounds. Using range-coded responses, it is calculated using the average informativeness score (INF) of Hemming et al., (2018): Eq. 7 as follows:

$$\mathrm{INF} = \frac{1}{N}\sum_{i=1}^{N} \left| \frac{w_i}{w_{\mathrm{i,max}}} \right| \tag{10}$$

  where w is width of the interval defined by the confidence interval in the classical model and by a given cut in the possibilistic approach (i.e. the 0.05-cut when the quantile at 5 and 95% are given in the classical approach); $w_{\max}=u-l$;

- *Calibration* is intuitively understood as the degree to which the uncertainty bounds contained the truth as often as specified; e.g. the expert's 80% intervals should capture the truth 80% of the time. It is calculated as the number of times, over the questions, the true answer falls within the bounds of interval defined by the confidence interval in the classical model and by the cut in the possibilistic approach. Note that we preferably analyse one minus the calculated number of times so that all three afore-described criteria are expected to be minimized. It is denoted HIT.

4.2.2 Results of BEMUSE programme (Nuclear Energy Agency)

This application test case is selected because it belongs to the application domain of interest for NARSIS project, namely nuclear safety analysis. The dataset of expert knowledge is provided by Destercke and Chojnacki (2008) within the BEMUSE (Best Estimate Methods - Uncertainty and Sensitivity Evaluation) programme performed by the NEA (Nuclear Energy Agency). The focus is here on the uncertainty analysis with experimental data coming from the experiment L2-5 performed on the loss-of-fluid test (LOFT) facility.

The dataset is composed of four seed questions about: 1. the first Peak Cladding Temperature during the blowdown phase (expressed in Kelvins), denoted 1PCT; 2. the second Peak Cladding Temperature during the reflood phase, denoted 2PCT; 3. the Time of accumulator injection (expressed in seconds), denoted T_{inj}; 4. the Time of complete quenching (expressed in seconds), denoted T_{q}. The panel is composed here of 10 experts. The experts are asked to provide the median and the percentiles at 5 and 95% (see Figure 12).

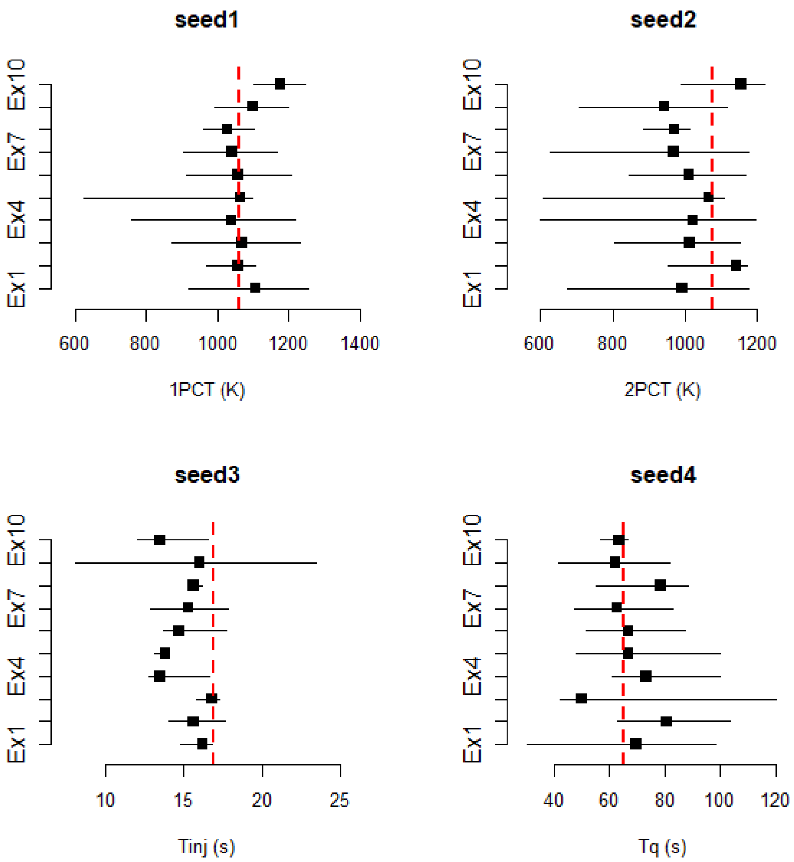

Figure 12: Answers to the four seed questions – BEMUSE test case

Training set analysis

Figure 13 provides the different scores. Several observations can be made:

- The best calibrated/accurate expert differs for both approaches (expert 2 and 3 for the classical model and the possibilistic method respectively);
- There is a good agreement regarding the worst calibrated/accurate expert (identified by both approaches as expert 8);
- There is a good agreement regarding the best and worst informative expert (identified by both approaches as expert 8 and 9 respectively).

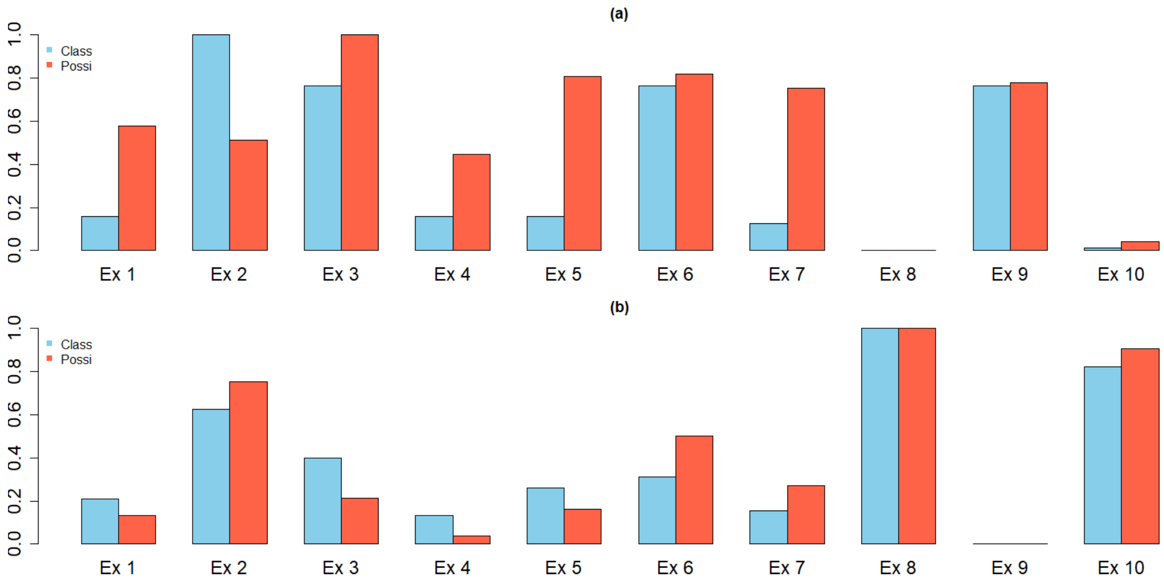

Figure 13: (a) Calibration scores (scaled between 0 and 1) with the classical model (blue) and the possibilistic method (red); (b) Informativeness scores (scaled between 0 and 1) considering the application test case for nuclear safety.

Figure 14 provides some additional elements to better explain the differences with respect to the calibration score. Fig. 14a shows the number of times the true answer falls within the inter-percentile interval for expert Ex 2 and Ex 3. The horizontal red lines indicate the values of the theoretical distribution (0.05 for inter-percentiles [0, 5%] and [95, 100%], and 0.45 for inter-percentiles [5,50%] and [50,95%]). This shows that the distribution of Ex. 2's answers is closer to the theoretical one than the one for Ex. 3. This explains the larger Cooke's calibration score for Ex 2. On the other hand, Fig. 14b shows that possibility degrees of Ex. 3 are mainly larger than the ones of Ex 3. In average, the possibility approach thus indicates Ex. 3 as the most accurate expert.

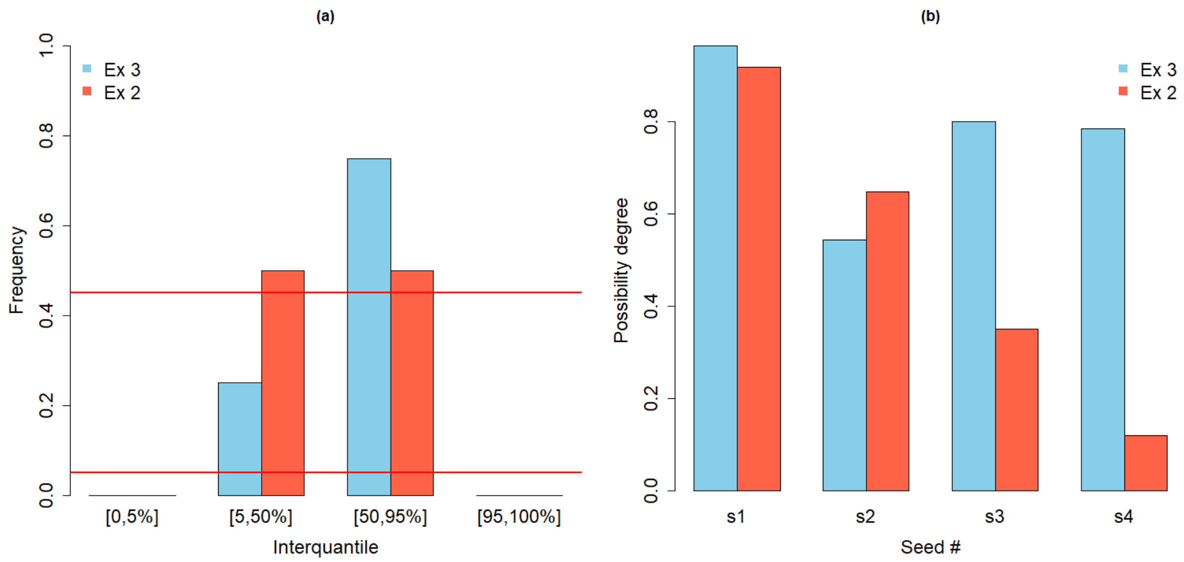

Figure 14: (a) Number of times the true answer falls within the inter-percentile intervals considering the Ex. 2 and 3; (b) Comparison of the degrees of possibility for each of the four seed variables for Ex. 2 and 3.

Test set analysis

The leave-one-out procedure is applied to the dataset. Figure 15 provides the performance criteria for the different aggregation methods (see description in Sect. 4.2.1).

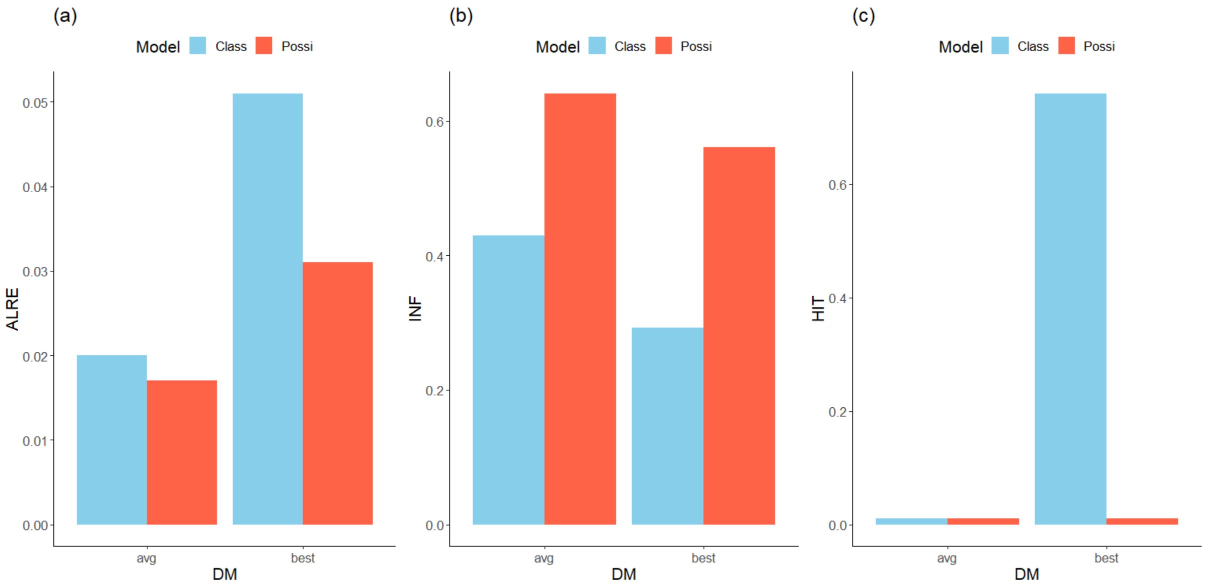

Figure 15: (a) Accuracy (ALRE) scores considering the classical model (blue) and the possibilistic method (red) given different ways of aggregating the expert opinions; (b) Informativeness (INF) scores; (c) Calibration (HIT) scores for the application test case of nuclear safety.

Several observations can be made:

- Choosing the best DM, whatever the approach, leads to the worst (maximum) ALRE (Fig. 15a), i.e. the least accurate expert;
- Only small ALRE differences exist between the classical and possibilistic approach when averaging is conducted;
- In general, the classical approach has the most informative answers (Fig. 15b);
- DM_{bestC} is the most informative (Fig. 15b) expert, but its calibration score HIT is the largest hence indicating misspecified answers (i.e. some true answers fall outside the provided interval q5-q95), Fig. 15c;
- Except for DM_{bestC}, all considered approaches achieve to cover all true answers.

4.2.3 Earthquake hazard assessment

This application test case has been selected in the application domain of interest for NARSIS project, namely probabilistic seismic-hazard assessment. The expert dataset is provided by Gerstenberger et al. (2016) with an application for Canterbury, New Zealand. The dataset is composed of 14 seed questions[4].

An example of questions is "Q13: In a typical PSHA model, the background smoothed seismicity model is based on a catalogue where clusters have been removed (declustered). In New Zealand, the Reasenberg declustering algorithm is employed to decluster the earthquake catalogue used for creating the background smoothed seismicity model (of the seismic hazard model). Reasenberg is a standard methodology that uses small earthquakes to link clusters of earthquakes together. Wellington is a moderate-to-high seismicity region in NZ but has experienced no large and extended aftershock sequences during the historical times used to create the background seismicity model. Please provide your estimated range

[4] http://www.seismosoc.org/Publications/SRL/SRL_87/srl_87-6_gerstenberger_et_al-esupp/SRL-2016084_esupp_Calibration_Questions.pdf

(10th, 50th and 90th percentile) for the percent change in the predicted 10% in 50 year PGA for Wellington when using a catalogue that contains all events and has not been declustered vs. one that has been declustered (the mean value and bounds may be either + or -)".

A panel of 12 experts is considered. The experts are asked to provide the median and the quantiles at 10 and 90%. Examples of answers are provided in Figure 16 for four seed variables: 1. Subducted distance *D*; 2. Rock uplift during earthquake event *U*; 3. Return period *RP* of an event with magnitude superior to 6; 4. Global average rate of events *R*.

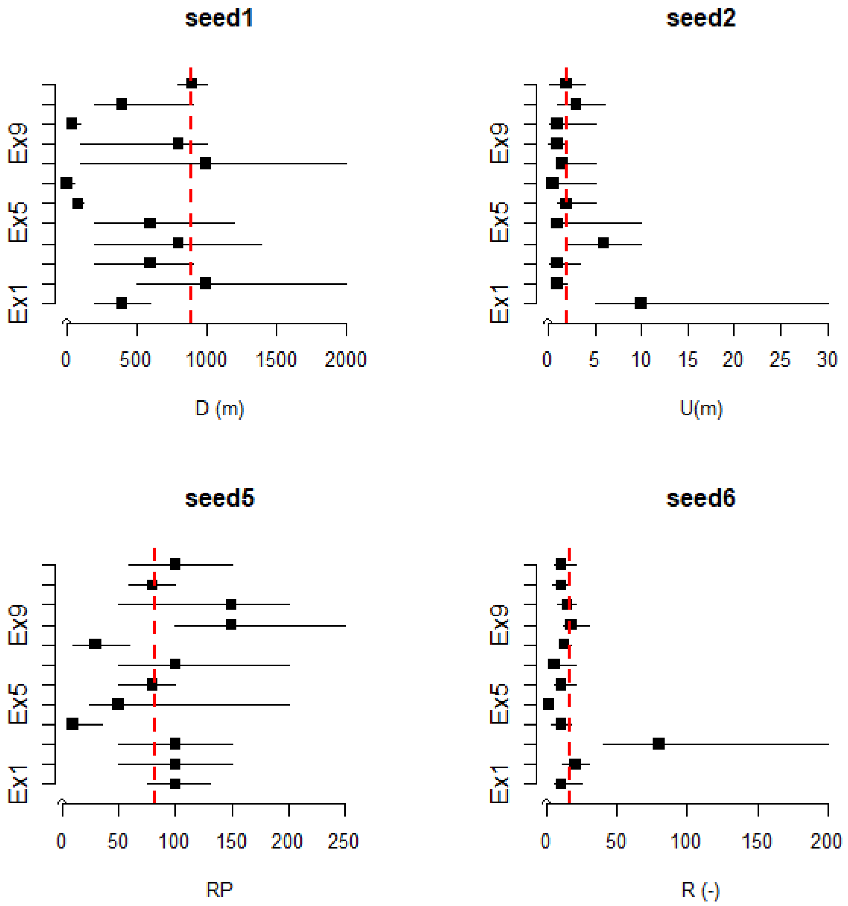

Figure 16: Examples of answers to the four seed questions – earthquake test case

Training set analysis

Figure 17 provides the different scores. Several observations can be made:

- The best calibrated/accurate expert differs for both approaches (expert 7 and 5 for the classical model and the possibilistic method respectively), though it should be noted that the differences in the score values for expert 5 are small;
- There are some differences regarding the worst informative expert (expert 10 and 2 for the classical model and the possibilistic method respectively), but the differences in the score values remain small.
- The classical approach appears to better discriminate the informativeness of the expert compared to the possibilistic approach considering the differences in the score value between the first and second most informative expert;
- There is a lack of agreement in the identification of the least informative expert.

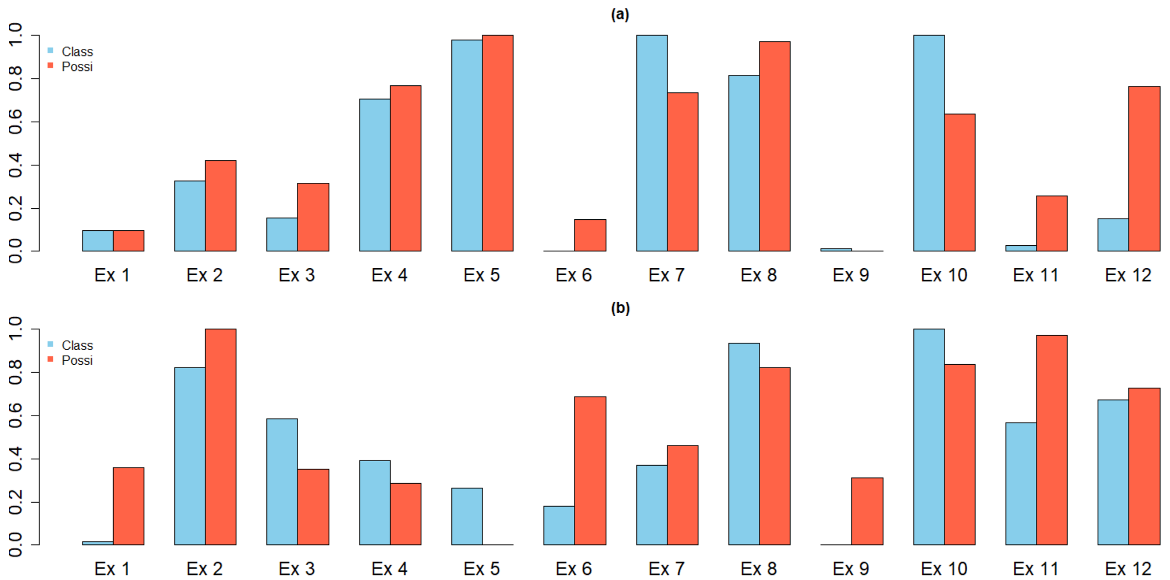

Figure 17: (a) Calibration scores (scaled between 0 and 1) with the classical model (blue) and the possibilistic method (red); (b) Informativeness scores (scaled between 0 and 1) considering the application test case for earthquake hazard assessment.

To further explain these differences, we provide additional analysis. Figure 18a shows the number of times the true answer falls within the inter-percentile interval for expert Ex 5 and Ex 7. The horizontal red lines indicate the values of the theoretical distribution (0.10 for inter-percentiles [0, 10%] and [90, 100%], and 0.40 for inter-percentiles [10,50%] and [50,90%]). This shows that the distribution Ex. 7's answers is slightly closer to the theoretical one than Ex. 5. The differences in the degrees of possibility is more straightforward. Fig. 18b shows that possibility degrees of Ex. 5 are mainly larger than the ones of Ex 7. In average, the possibility approach thus indicates Ex. 5 as the most accurate expert.

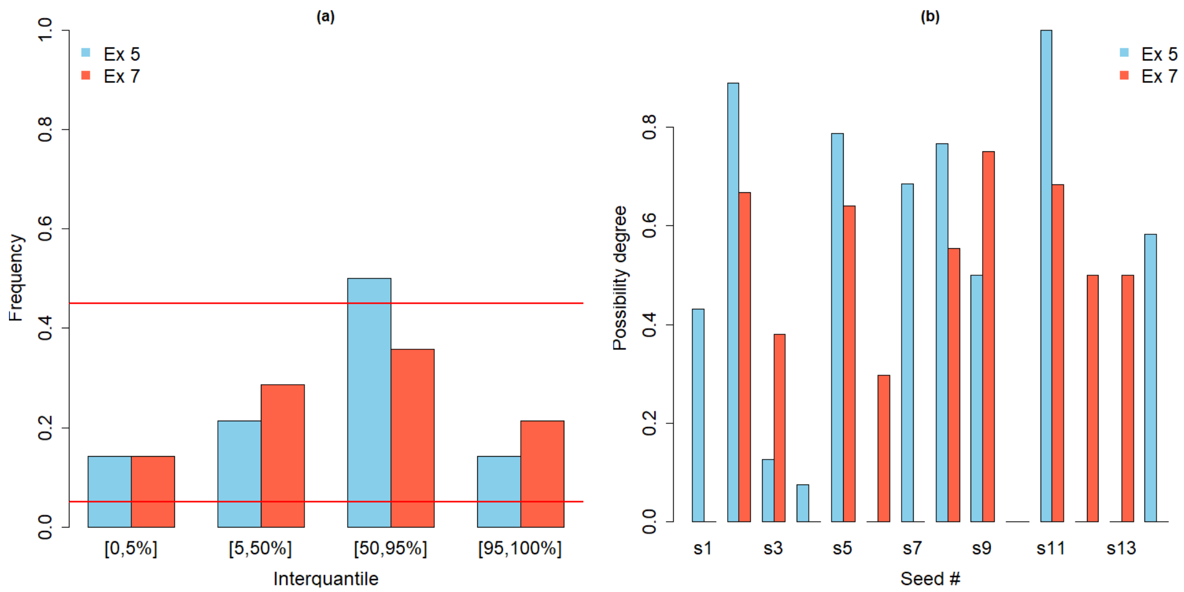

Figure 18: (a) (a) Number of times the true answer falls within the inter-percentile intervals considering the Ex. 5 and 7; (b) Comparison of the degrees of possibility for each of the four seed variables for Ex. 5 and 7.

Test set analysis

The leave-one-out procedure is applied to the dataset. Figure 19 provides the performance criteria for the different aggregation methods (see description in Sect. 4.2.1).

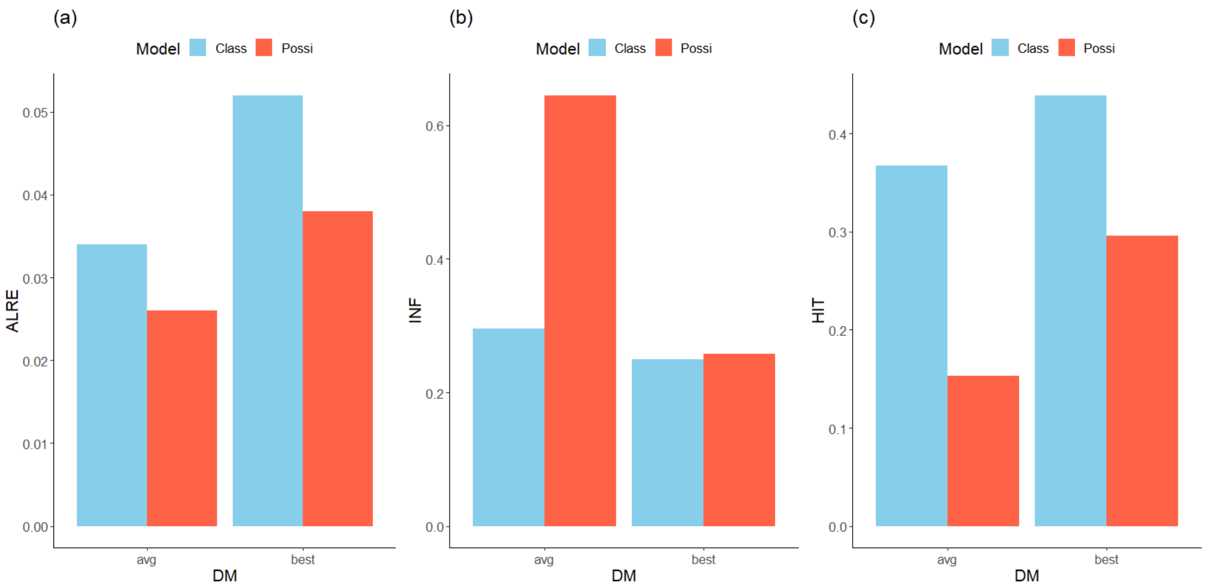

Figure 19: (a) Accuracy (ALRE) scores considering the classical model (blue) and the possibilistic method (red) given different ways of aggregating the expert opinions; (b) Informativeness (INF) scores; (c) Calibration (HIT) scores for the application test case of earthquake hazard assessment.

Several observations can be made:

- Choosing the best DM, whatever the approach, leads to the worst (maximum) *ALRE* value (Fig. 19a);
- Here DM_{avgP} leads to the minimum *ALRE*, i.e. the most accurate expert;
- DM_{best} is the most informative (Fig. 19b) whatever the approach;
- In general, the possibilistic intervals cover the best the true answers, i.e. the HIT criterion is the lower (Fig. 19c).

4.3 Summary and recommendations

In this section, we have investigated the benefit of using an alternative to the classical model of Cooke (1991) for evaluating expert-based information, i.e. an approach based on the possibility theory.

From a practical point of view, the criteria of calibration and informativeness in the frame of the possibility theory (Dubois and Prade, 1988) are built with very intuitive notions, namely the width of the uncertainty range and on the distance between experimental and reference calculations. On the contrary, the classical model uses statistical concepts to translate such criteria using statistical distance, i.e. KL divergence, whose validity depends on the number of seed variables (see the extensive cross validation exercise by Eggstaff et al., 2014). Besides, the computation of the possibilistic criteria is straightforward and can easily be integrated in any safety analysis, because they simply derive from the graphical analysis of the possibility distributions (see Figures 10 and 11).

The two-stage comparison highlights the following aspects.

Regarding the problem of expert ranking (*training set analysis*),

- Both examples show that the ranking can be completely different; especially for the calibration score;

- These differences should not be interpreted as a flaw in any of the approach: they are expected given the different focuses of each criterion. This can be considered a practical recommendation:

    - when the focus is on measuring the deviation from a reference value (i.e. a best estimate), the possibilistic calibration score is more appropriate;
    - when the focus is mainly on the statistical distribution of seed values in relation to the inter-percentiles given by the expert, the classical model is more appropriate (i.e. when experts provide 5-95% interval, we expect the expert's answer to fall 90% of the times within this interval).

- The differences can be explained (Figures 14, 18), and searching the reasons for the dissimilarities can improve the evaluation analysis. Being aware that both approaches provide two viewpoints on the problem, it could envisage to mix these two pieces of information or leave it to the decision-maker to decide, which assessment best serves her/his particular interests.

Regarding the problem of prediction (*test set analysis*),

- Whatever the approach, there is a benefit of relying on the result derived from the averaging of the expert-based information compared to relying on the information expert with best ranking score;

- The higher accuracy of the predictions is achieved by using the possibilistic approach. This confirms the conclusions relative to differences in the ranking using the different calibration scores, i.e. the possibilistic calibration score is by construction focused on the notion of distance, and is expected to achieve a low *ALRE* score;

- The higher accuracy of the possibilistic approach appears however to come at the expense of higher imprecision (less informative);

- Using the classical model is however riskier (higher HIT score), because we are not ensured to find the true values within the interval derived from the averaging aggregation.

These conclusions should be strengthened regarding:

- The confrontation to a larger number of test cases by taking advantages of the TU expert judgment database (Cooke and Goossens, 2008) with respect to both aspects of the problem, i.e. ranking and prediction;

- The use or adaptation of more complex aggregation methods (see for example, Delmotte 2007, Dubois and Prade 2001) to allow for a finer analysis and treatment of conflicting information.

5 Expert-based modelling using Fuzzy Expert Systems

This section deals with the use of fuzzy expert systems to model the expert knowledge and to reproduce expert-like reasoning. In the following, we provide the key ingredients of fuzzy logic (Sect. 5.1), namely the procedure for knowledge modelling using fuzzy constraints (Sect. 5.2) and for performing the reasoning using fuzzy rules (Sect. 5.3). Two types of fuzzy expert systems relevant for NARSIS project are introduced in Sect. 5.4, and details on how to conduct fuzzy knowledge management in practices are provided in Sect. 5.5. Finally, we analyse how fuzzy expert systems can be transposed to NARSIS project for risk assessment of nuclear power plants. On this basis, we formulate some recommendations.

5.1 Fuzzy logic

Fuzzy logic was introduced by L.A. Zadeh in 1965 (Zadeh 1965) in order to get closer to the human reasoning and to avoid the threshold effects pertaining to classical logic approaches. Indeed, common sense knowledge, which is by nature both lexically imprecise and non-categorical, is not correctly handled by classical formalisms. The development of fuzzy logic has thus been motivated by the need of a conceptual framework that can address uncertainty and lexical imprecision (Dubois & Prade, 1995). Fuzzy logic must be viewed as an extension of classical logic: whereas classical logic deals only with two values (true and false), fuzzy logic considers an infinite number of real values between 0 and 1. Hence, exact reasoning is a particular case of approximate reasoning.

Fuzzy logic is based on the concept of fuzzy sets (Zadeh, 1996, 1999). Let X be the universe of discourse, i.e. a collection of objects denoted x. A fuzzy set A of *X* is totally characterized by a function as follows:

$$\mu_A : X \to [0;1]$$

called membership function. This function indicates how much an object x belongs to *X*. In crisp sets, an object does or does not belong to the set. Fuzzy sets are useful to describe imprecision or uncertainty. Figure 20 represents a fuzzy set defines on [0, 10] with a triangular membership function and which can symbolize “approximately 5”. Membership functions can have different shapes (e.g. instance triangular, trapezoidal, Gaussian, or Bell functions).

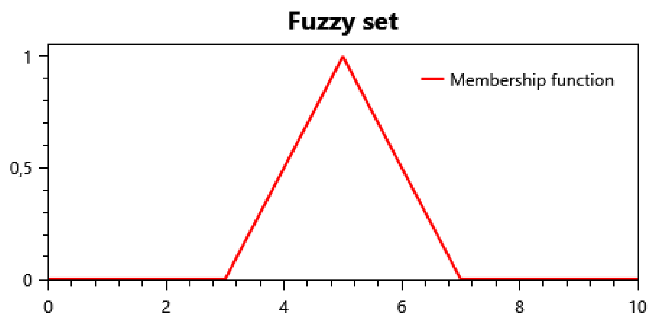

Figure 20: Example of fuzzy set

The next concept in fuzzy logic, called linguistic variable (Zadeh, 1974) corresponds to a triplet (V, X, T_V) where:

- V is the name of the linguistic variable,
- X is the domain on which it is defined (e.g. R or [0,10]),
- $T_V = \{T_1, T_2, \ldots\}$ is a finite collection of fuzzy sets called terms which qualify V (e.g.: “cold”, “hot”).

Each fuzzy set T_i of T_V is associated with a name and is called linguistic term. Figure 21 shows a linguistic variable called "temperature" with three terms ("cold", "medium", "hot") and their membership functions, defined on [-20, 50].

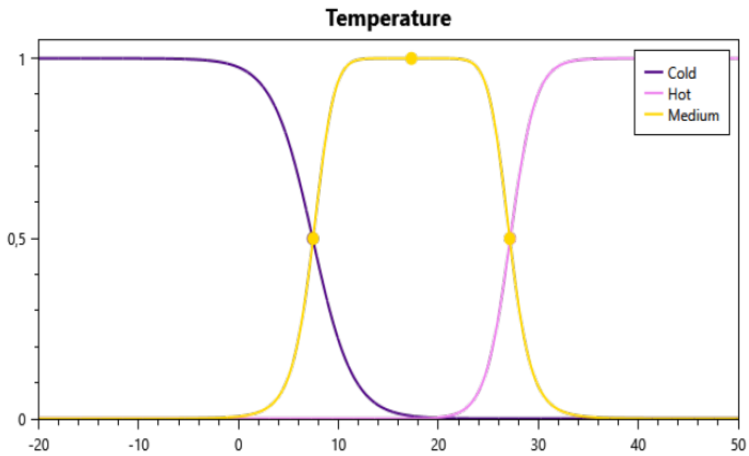

Figure 21: Example of a fuzzy linguistic variable "Temperature"

Some of the main differences between fuzzy logic and traditional logic are stated after (Dubois et al., 2000):

- In bivalent logic, truth can have two values: true or false. In fuzzy logic, the truth value of a proposition is an element of the interval [0,1].
- In bivalent logic, predicates are crisp, e.g. even, larger than…. In fuzzy logic predicates are, for example, tall, soon, much larger than.
- Classical systems use only one predicate modifiers, i.e. not. In fuzzy logic, one can use very, more or less, etc.
- Classical logic introduced two quantifiers: existential and universal. Fuzzy logic admits also few, several, most, etc.

The importance of fuzzy logic comes from the fact that there are many real world applications that fit these conditions (Harris, 2000), especially in the domain of knowledge-based systems (DuBois & Prade, 2011) for decision-making (Bellman & Zadeh, 1970) and control (Zimmermann, 2014).

Fuzzy logic allows different types of reasoning (the examples are taken from (Zedeh, 1989)):

- Categorical reasoning:

  Ex.:

  Carol is slim
  Carol is very intelligent

  ➔ Carol is slim and very intelligent

  Slim, *very intelligent* are predicates and the conclusion is a conjunction. Premises cannot contain quantifiers.

- Syllogistic reasoning:

  Ex.:

  Most Swedes are blond
  Most blond Swedes are tall

  ➔ $Most^2$ Swedes are blond and tall

  Here premises contain quantifiers. *$Most^2$* is the square of the fuzzy set *most* in fuzzy arithmetic.

- Dispositional reasoning:

Ex.:

Heavy smoking is a leading cause of lung cancer

➔ To avoid lung cancer avoid heavy smoking

In this case, premises are propositions that are preponderantly but necessarily always true.

- Qualitative reasoning:

  Ex.:

  If pressure is high then volume is small

  If pressure is low then volume is large

  ➔ If pressure is medium, volume is (w_1 ^ small + w_2 ^ large)

  In this case, w_1 and w_2 are coefficients that represent the degree to which the premises are true.

The last type of reasoning is the most used, in particular in fuzzy systems.

5.2 Knowledge modelling

Knowledge can be viewed as a collection of propositions that must be semantically defined. In fuzzy logic, meaning representation is based on test-score semantics: a proposition in natural language must be viewed as fuzzy constraints (Zadeh, 1989). For example, *temperature is high* represents an elastic constraint on the temperatures. Representing the meaning of a proposition *p* through test-score semantics consists in three steps:

- Identification of the variables whose values are constrained by *p*;
- Identification of the constraints induced by *p*;
- Characterization of each constraints by describing a procedure which associates to each constraints a number from [0, 1];
- Aggregation of the partial scores into one score.

For instance, if the proposition is “Jean is tall and blonde”, the variables are his height and his hair color; the constraints are *tall* and *blonde* that are characterize by the membership functions of each fuzzy set; finally, the aggregation is performed by the conjunction.

To manipulate those test-scores, three categories of rules can apply:

- Composition: if test-scores of two elastic constraints C_1 and C_2 respectively equal v_1 and v_2, then

  - C_1 and C_2 is $v_1 \wedge v_2$ (conjunction)
  - C_1 or C_2 is $v_1 \vee v_2$ (disjunction)
  - If C_1 then C_2 is $1 \wedge (1 - v_1 + v_2)$ (implication)

- Modification: if test-score for an elastic constraint C_1 is v then
  - Not C is 1 – v (negation)
  - Very C is v^2 (concentration)
  - More or less C is $v^{1/2}$ (diffusion).

- Quantification: the propositions are of the form Q A’s are B’s where Q is a quantifier and A and B are fuzzy sets, like in “Most of the Swedes are blond”, and use the sigma-count to qualify these constraints.

Previous definitions were used to formalize human reasoning through a set of rules (Aliev & Tserkovny, 2011). These rules depend on a combination of elementary fuzzy propositions. An elementary fuzzy proposition is the definition of “V is A” from a linguistic variable (V, X_V, T_V) where A is a term of T_V. For instance, if one used the linguistic variable “temperature” of Figure 21, a fuzzy proposition could be “temperature is cold”. It is evaluated from its membership function μ_{cold}: for a particular temperature t, the truth-value of the proposition “temperature is cold” is given by $\mu_{cold}(t)$. The truth-value of a fuzzy proposition belongs to the interval [0, 1] in R unlike classical logic where the truth-value of a proposition would be either 0 (false) or 1 (true).

A fuzzy expression stands for a composition of a set of elementary fuzzy propositions or other fuzzy expressions. The composition is created by selecting logical operators: negation (not), conjunction (and) and disjunction (or).

Fuzzy propositions can be viewed as a special case of fuzzy expressions. For instance, let two fuzzy propositions be “V is A” using (V, X_V, T_V) and “W is B” using (W, X_W, T_W). Thus, “V is A and W is B”, “V is A or not W is B” are fuzzy expressions. The truth-value of a fuzzy expression is given by the application of all the operator functions on the values of their operands.

There are different formulations of conjunction and disjunction operators, but they all have to fulfill the properties of respectively triangular norms and triangular co-norms. A triangular norm (t-norm) is a commutative, associative, monotonic function $\top: [0;1] \times [0;1] \rightarrow [0;1]$ which accepts 1 as neutral element. A triangular co-norm (t-conorm) is a commutative, associative, monotonic function $\perp: [0;1] \times [0;1] \rightarrow [0;1]$, which accepts 0 as neutral element. For instance, Zadeh's norm is the minimum function and Zadeh's co-norm is the maximum function.The negation operator is a function $N: [0;1] \rightarrow [0;1]$, which generally associates to a fuzzy value n the value 1-n.

Fuzzy predicates are sometimes called fuzzy relations. The literature counts many papers about fuzzy relations in different dimensions.

Temporal relations like *before*, *after*, *meanhile*, have been introduced in different ways regarding the type of events that are described. Dubois and Prade (DuBois & Prade, 1989; Dubois et al., 2003) introduced fuzzy dates and offer relations between them. Schockaert (Schockaert et al., 2008; Schockaert & Cock, 2008) fuzzified Allen’s relations based on intervals comparison. Other authors have proposed to collect events on a fuzzy scope and defined operators (Poli et al., 2016) such that occurs, persists, etc. All those relations allow making decision based on the temporality of a phenomenon.

In the spatial domain, Schockaert (Schockaert et al., 2009) propose a fuzzy definition for RCC-8 spatial relations that allow describing the relation between two spatial reasons. Bloch (Bloch, 2005) use fuzzy morpho-mathematics to introduce a framework for fuzzy spatial reasoning (Vanegas et al., 2010) (Vanegas et al., 2011). Such relations car describe situations for which the spatial aspect is important. It can be coupled with a Geographic Information System to perform geo-spatial fuzzy reasoning.

Spatio-temporal relations (Yaouanc & Poli, 2012) (Poli et al., 2018) allow describing the move of an object and its changes of shape. Few spatio-temporal relations have been introduced to monitor vehicles and determine if they are moving, crossing an area, etc.

Fuzzy expert systems are a particular case of fuzzy logic systems whose knowledge is modeled as fuzzy rules.

5.3 Fuzzy rules

The expressions are used to express the knowledge of an expert as fuzzy rules. A fuzzy rule is composed of a premise (also called antecedent) and a conclusion (also called consequent) in the form “IF premise THEN conclusion” (Dubois & Prade, 1996) (Dubois, Prade, 2003).

A premise is a fuzzy expression and a conclusion is a fuzzy statement which value can be of different nature. There are two different types of rule conclusions and fuzzy rules:

- A conclusion can be a fuzzy proposition. Then, the template of a fuzzy rule becomes:

  “IF temperature is low THEN sweating is low”.

- A conclusion can be a mathematical function of the inputs of the premise. These rules are called Takagi-Sugeno fuzzy rules and their template is:

"IF temperature is low THEN sweating=f(temperature)".

The rules are mapping the inputs to the outputs of the system regarding a fuzzy implication function.

The THEN part of fuzzy rule denotes a fuzzy implication function I such as I:[0, 1]→[0, 1] that performs on two fuzzy values. It verifies the properties of the implication operator in classical logic when the two operands equal 0 or 1.

The implication operator in classical logic states that either we do not have the antecedent or we have the consequent. There are different formulations of fuzzy implication function inspired from multi-valued logic. Some conjunctive functions, which make it possible to use both inputs and outputs, can be used instead of the implicative functions. That is the case for the well-established Mamdani fuzzy rules that make use of the minimum function.

This kind of rules is the most frequent and is often see as the basic unit for knowledge in fuzzy logic. However, there exist other types of rules that come from either fuzzy logic or possibility theory (Ughetto et al., 1999):

- Certainty rules: *the more X is A, the more certainly Y lies in B*
- Gradual rules: *the more X is A, the more Y is B*
- Possibility rules: *the more X is A, the more possible Y lies in B*
- Antigradual rules: *the more X is A, the larger the set of possible values for Y is, around the core of B*
- Etc.

5.4 Fuzzy expert systems

Now that the basics of fuzzy logic have been introduced, we can focus on the process of inference in fuzzy expert systems. In the previous section, we introduced how rules work on their own. In this section, we present how rules work together to compute the outputs and thus process the inference in a fuzzy expert system.

A fuzzy expert system (Nagori & Trivedi, 2014) is composed of several main components (Figure 22):

- As classical expert systems, a knowledge base and an inference engine;
- A fuzzifier and defuzzifier that are specific to fuzzy expert systems and that allow turning crisp values into fuzzy values and inversely.

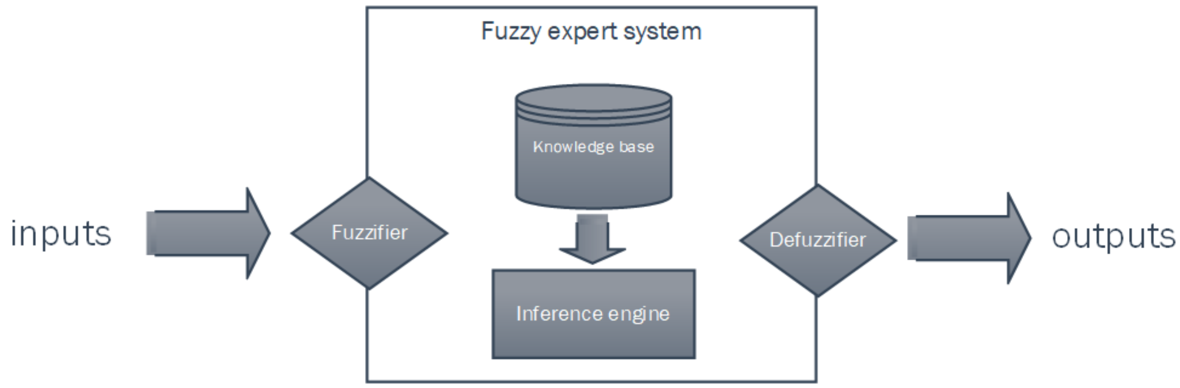

Figure 22: Overview of a fuzzy expert system

As classical expert systems, fuzzy expert systems are good candidates to be an Explainable Artificial Intelligence thanks to their handling of natural language and the way they infer. It is possible to generate a textual explanation of decisions (Baaj & Poli, 2019).

5.4.1 Mamdani inference

In this section, we will describe Mamdani inference process (Mamdani & Assilian, 1975), summarized in Figure 23.

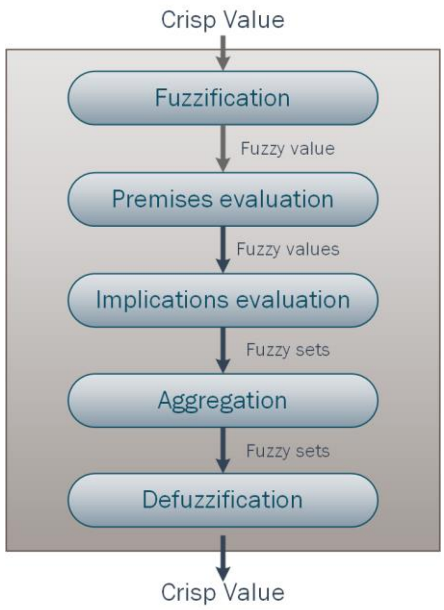

Figure 23: Mamdani inference.

The first step consists in the evaluation of each elementary fuzzy proposition in the premise of the rules: this step is called *fuzzification*. It takes a crisp value (the value of the input in the real world) and associates a fuzzy value to it regarding the membership function of the term of the linguistic variable involved in the fuzzy proposition. For instance, with the linguistic variable "temperature" of Figure 21 and its term "cold", for a temperature of -10°C, "temperature is cold" is evaluated at 1 (i.e. -10°C is really a cold temperature). For a temperature of 8°C, both "temperature is cold" and "temperature is medium" equal 0.5 (approximately).

The premises are then computed with the application of the t-norms, t-conorms, negation function and other operators according to the rules involved. The value of each premise is a fuzzy value also called rule activation. For instance, considering Zadeh's t-conorm (i.e. the max function) and a temperature of 20°C, the value of the premise "temperature is cold or temperature is medium" equals max(1,0)=1.

The implication function is then applied on the conclusion of each rule. The result is a fuzzy set describing the output. For instance, if the premise of the rule has been evaluated at 0.75, with the implication method being the minimum function, the output's fuzzy set is as shown in Figure 24.

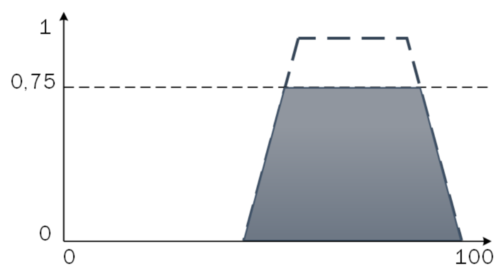

Figure 24: Result of the implication

The diverse resulting fuzzy sets of a same output are aggregated by an aggregation function, often the max function for Mamdani systems, resulting in a fuzzy set for each output. For instance, Figure 25 illustrates the process of aggregation of two fuzzy sets with the max aggregation function.

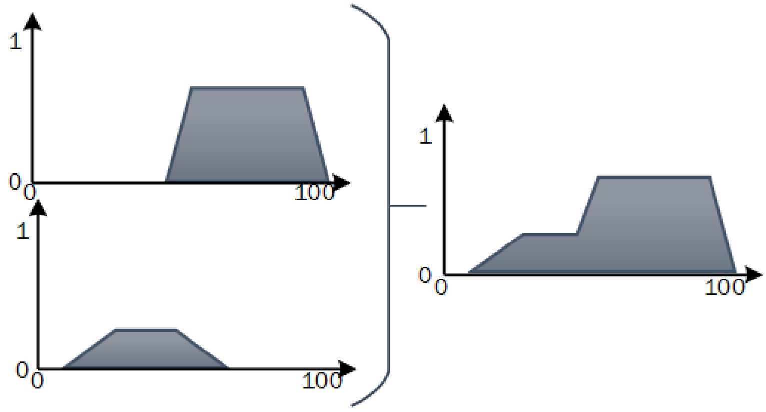

Figure 25: Application of the max aggregation on two fuzzy sets

Each output's fuzzy set is changed into a crisp value: this process is called *defuzzification*. There are several defuzzification methods; the best-known defuzzification method computes the centroid of the area under the membership function (the blue area of the aggregated fuzzy set in Figure 25). Figure 26 shows on the same fuzzy set resulting from the aggregation phase the different output values that are obtained with the different defuzzification methods.

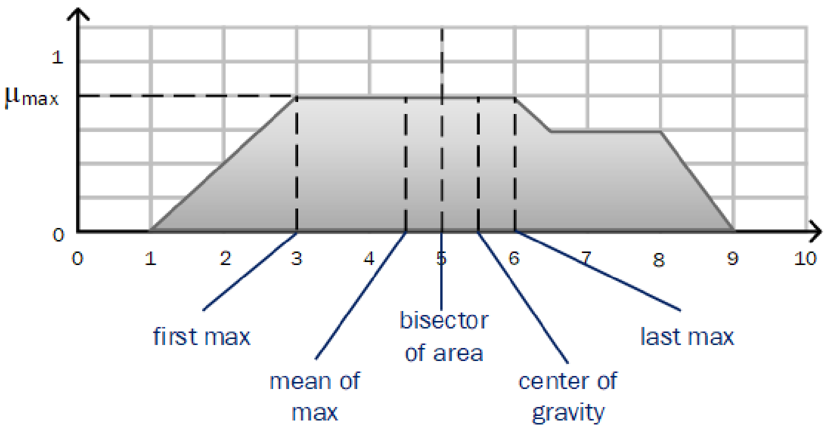

Figure 26: Differences between the various defuzzification methods

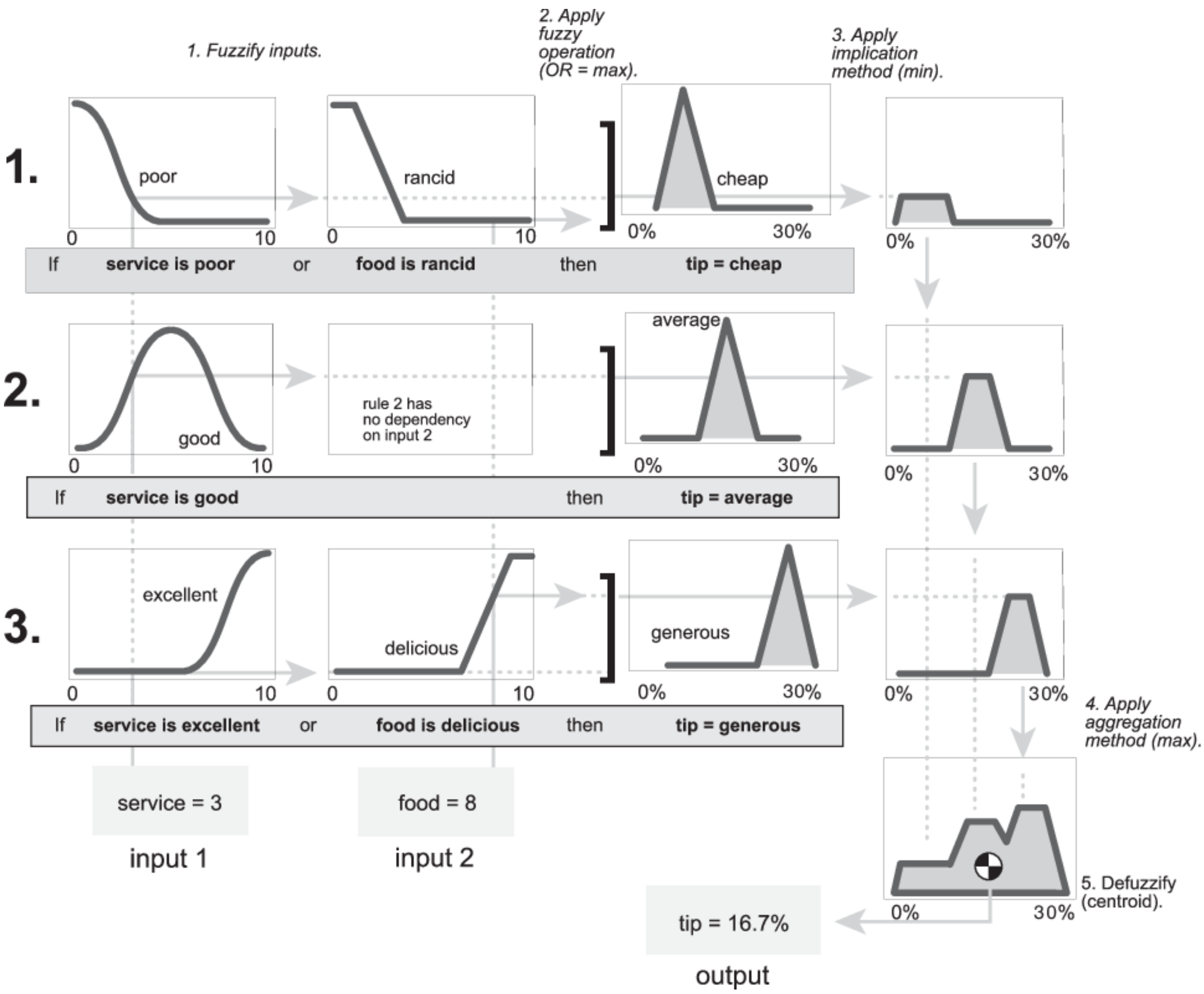

Figure 27: Comprehensive example of tip problem with Mamdani inference (Mathworks, 2020)

Figure 27 shows a comprehensive example of a Mamdani fuzzy inference system for toy problem of tip recommendation. The goal of this problem is to assess the tip in a north-american restaurant, between 0% and 30% of the total bill, regarding the quality of food and of service, given with a scale between 0 and 10.

5.4.2 Sugeno inference

The Sugeno inference (Sugeno, 1985) differs from the Mamdani inference by the way the conclusion are treated and the rule are aggregated. In this system, each rule computes a defuzzified value that are aggregated then. The overview is shown in Figure 28.

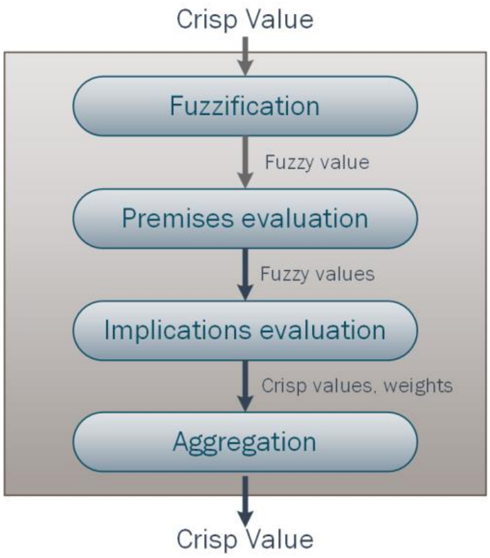

Figure 28: Sugerno inference overview

The premise evaluation is still the same. Each rule produces two values: a crisp value that is obtained by applying a function (often linear) to the input values of the rule, and a weight that is the activation value of the rule.

Then a weighted sum is perform over the rule outputs in order to obtain only one value.

Sugeno inference is known to be more effective in terms of computations than Mamdani inference. However, the definition of the outputs is less intuitive than in Mamdani rules.

Figure 29 shows a comprehensive application of Sugeno systems for the tip problem introduced before.

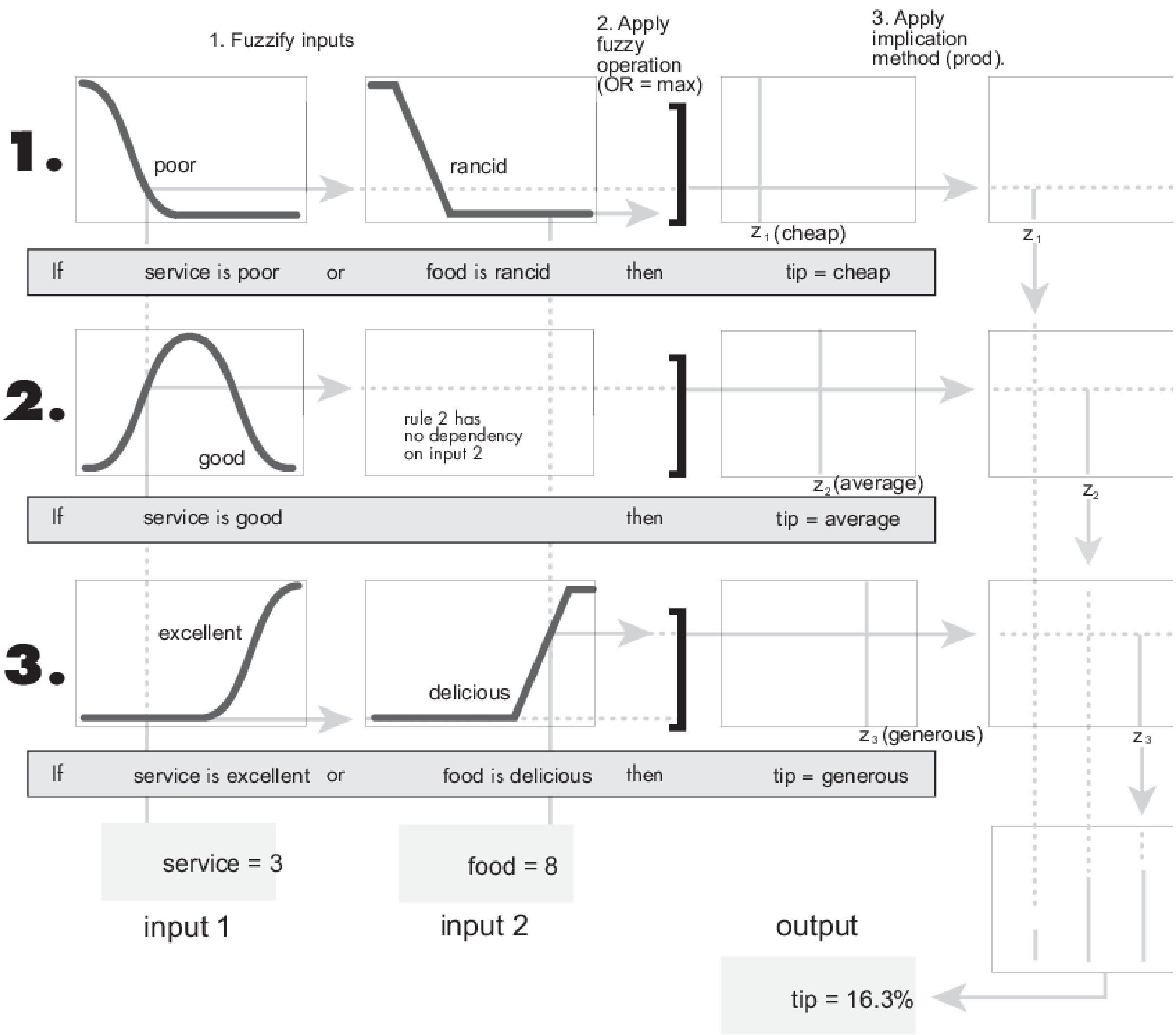

Figure 29: Comprehensive example of tip problem with Sugeno inference (Mathworks, 2020)

5.5 Tools for fuzzy knowledge management

In this section, we describe how to collect the knowledge and formalize it with fuzzy logic.

The principal way to collect expert knowledge is by interviews. Different methods can be applied to that task that still is the tedious part.

The MKSM (Methodology for Knowledge System Management) method was designed in 1992 by Ermine to manage (save and capitalize) the researchers' knowledge (Ermine et al, 1996). The following year, it was set up at CEA (French Atomic Energy Commission). It was then enriched to become MASK (Method for Analysis and Structuring Knowledge) (Matta et al., 2002). Today, it is used in many companies in France and around the world such as EDF, Thomson, La Poste, Cofinoga, Storengy, etc.

The approach of MKSM / MASK does not consists in a single model, it consists in modelling knowledge from various points of view around "sources of knowledge" of the company (knowledge holders, experts, specialists ...) (Zhu et al., 2019).

Different tools have been developed to author rule bases provided by experts. The most efficient, yet limited, is called association matrix (Figure 30). A table is displayed, whose columns are values of a first linguistic variable and rows are values of a second one. Each cell allows selecting the term of a Mamdani output. The association matrix creates a complete rule base in few clicks when there are exactly two inputs and one output.

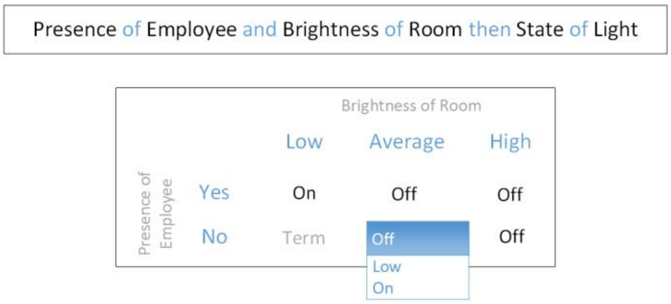

Figure 30: Association matrix

Another way to author rules is given by flow charts (Figure 31). They are appreciated by engineers but other end-users experience some difficulties with them.

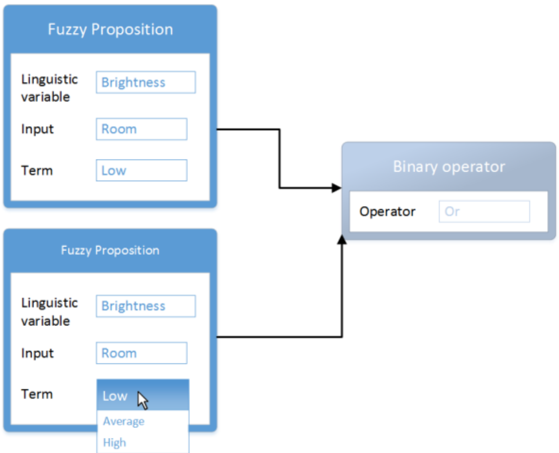

Figure 31: Partial flowchart for a disjunction

More advanced graphical user interfaces have been introduced. ExpressIF© (Poli & Boudet, A fuzzy expert system architecture for data and event stream processing, 2017), the fuzzy expert system developed by CEA, introduced a way to author rules by drag-and-drop and clicks interactions (Poli & Laurent, 2016), as shown in Figure 32.

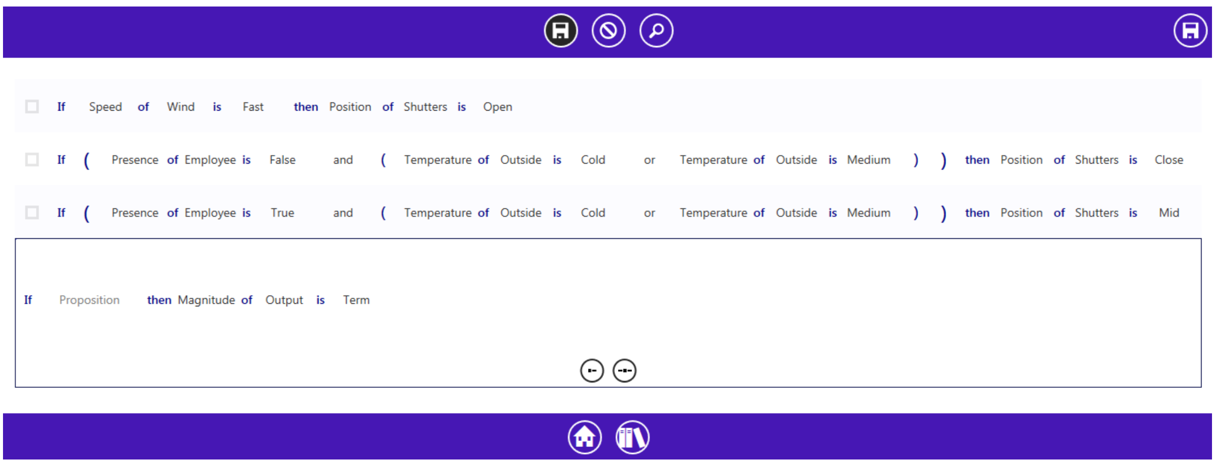

Figure 32: ExpressIF© drag-and-drop based rule editor

In scenario for which data are accessible, the fuzzy logic community has also proposed different algorithms to learn rule from data (Wang & Mendel, 1992). Most of them consist in inducing fuzzy decision trees (Yuan & Shaw, 1995; Olaru & Wehenkel, 2003; Janikow, 1998; Sébert & Poli, 2018) or gradual decision trees (Marsala, 2012; Marsala & Rifqi, 2017). Other methods rely on association rules (Au & Chan, 2005; Pierrard et al., 2018) or data set coverture (Hühn & Hüllermeier, 2009; Pierrard et al., 2018). It is important to note that these algorithms can also work with few data (Pierrard et al., 2019).

Metaheuristics can also be used to induce a rule base (Cordón et al., 2001) or also to optimize the parameters of an existing rule base from expert interviews (Cabrita et al., 2006; Cazarez-Castro et al., 2010; Danyadi et al., 2010; Johanyak & Papp, 2011, 2012).

5.6 Application to risk assessment for nuclear power plants

In this section, we analyse how the afore-described approaches can be transposed / adapted for 1) risk assessment; 2) monitoring and diagnostic analysis of accidents of nuclear power plants.

5.6.1 Risk assessment

Risk can be defined as the existence of inherent uncertainties in systems that affect their normal behavior (Karimpour et al., 2014). Most of time, it is difficult to collect data to train a model. Fuzzy expert systems have thus being applied in different domains of risk assessment (Shang & Hossen, 2013; Chan & Wang, 2013): explosion (Markowski et al., 2011), pipelines (Karimpour et al., 2014; Boudet et al., 2018), marine safety (Jing-qi, 2006), but also for NPP-related analyses either for risk analysis of engineered systems (e.g., Guimarães and Lapa, 2007) or for human reliability (Baraldi et al., 2015). In many applications, Mamdani fuzzy expert systems are used to collect uncertain knowledge from experts and then compute a value to characterize the risk.

Other methods have also been proposed (Shapiro & Koissi, 2015):

- **Fuzzy risk matrix** (Weidon, 2014; Can & Toktas, 2018) is the second most used technique. It is based on the observation that risk is defined by likelihood of the occurrence of the event and the severity of the consequences. In the fuzzy version, membership functions are used to the severity, the likelihood and the risk itself. Such matrices are related to association matrices presented before (indeed there are exactly two inputs and one conclusion). It can then be converted in rules like IF likelihood is low and severity is high THEN risk is medium. An example of fuzzy risk matrix is shown in Figure 33: L stands for low, M for moderate, H for high and MH for moderately-high;

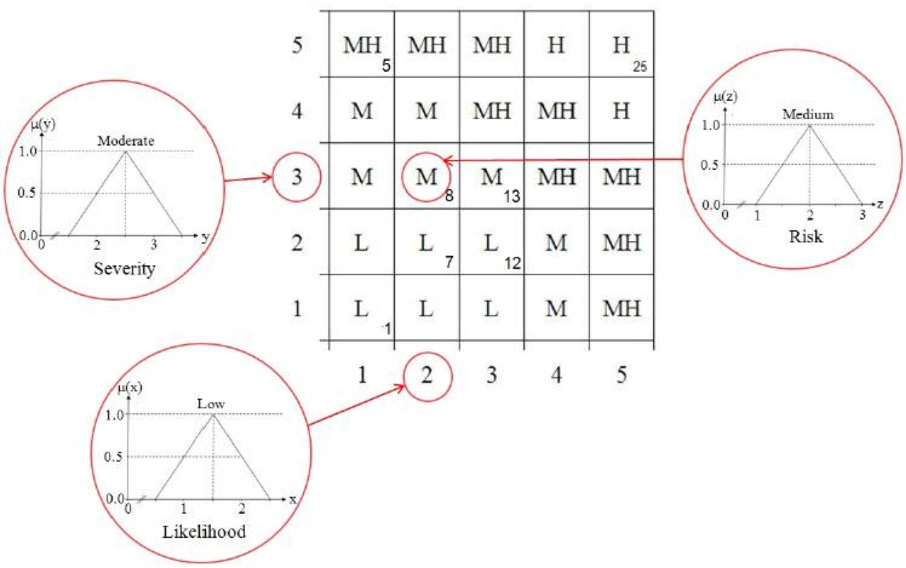

Figure 33: Example of fuzzy risk matrix (Shapiro & Koissi, 2015)

- **Fuzzy Analytic Hierarchy Process** (Buckley et al., 1999) is based on AHP that is a technique for making complex decisions, based on mathematics and psychology. It aims at weighting decision criteria. It gathers different experts opinions and performs pairwise comparisons. Each expert has to compare the relative importance between two criteria. The process is decomposed in a 3-level hierarchy: the goal of the decision at the top level, then the criteria, finally the alternatives. In the fuzzy counterpart, the pairwise criteria comparisons are fuzzy values. There are a multitude of fuzzy counterparts, that all have advantages and drawbacks;

- **Fuzzy Risk Priority Number** (Guimarães and Lapa, 2007; Guimarães et al., 2011) is derived from Risk Priority Number in Failure Mode and Effects Analysis with applications on nuclear engineering systems. It is a score that helps to sort recovery actions regarding:
  - Severity: failure severity is classically ranked in a scale from 0 to 10;
  - Occurrence: the potential of failure occurrence on the same scale;
  - Detection: the capability of failure detection also on the same scale.

  In the crisp version, the total score is just a multiplication of the three scores. In fuzzy logic, membership functions are used to characterize each axes and Mamdani fuzzy inference is used to assess the total score;

- **Fuzzy Fault Trees** are graphical models that are widely used in probabilistic safety assessment, especially in the domain of nuclear power plants. In these models, fuzzy logic brings a way to avoid precise failure probabilities or failure rates. Indeed, classical probabilities are replaced by fuzzy probabilities that are fuzzy numbers. (Purba, 2014) compares two different types of approaches using fuzzy-based approaches and hybrid approaches that mix fuzzy logic and classical probabilities.

Regarding the scope of NARSIS project, the following lessons drawn from existing studies are outlined:

1) Guimarães et al. (2011) conclude that the use of linguistic variables allow expert to describe the situations in more realistic ways;
2) Shin et al. (2016) compare AHP and fuzzy AHP on nuclear power plant construction. The authors conclude that fuzzy AHP overcomes the performances of its crisp counterpart;
3) In the comparison exercise between fuzzy expert systems and BBNs, Baraldi et al. (2015) highlighted that in cases characterized by very limited knowledge, an analyst

may feel constrained by the BBN probabilistic framework, and in these cases, the fuzzy expert systems can lead to a more transparent representation of the input and output uncertainty.

5.6.2 Accident and failure detection, monitoring and diagnostic

The use of expert systems for accident and failure detection, monitoring and diagnostic is not new in the domain of nuclear power plants. REACTOR was the first expert system for fault diagnosis in NPPs (Nelson and Blackman, 1987) that used a tree-based method. After REACTOR, TAMUS was designed for real-time identification of multiple failure in severe accidents (Martin and Nassersharif, 1988). Yet, high uncertainties, weak knowledge of severe accidents phenomena and inaccurate data from sensors during severe core damage are huge difficulties for classical approaches.

To handle these situations, fuzzy expert system is a valuable option that can bring insights into the continuous monitoring of the power plant of its subsystems. Fuzzy predicates, in particular fuzzy relations, are widely used for that purpose.

For instance, Poli et al. (2017) use temporal relations (similarly as done by Poli et al. (2016)) to monitor a wind turbine equipped of several sensors and tries to anticipate failure. They propose an approach based only on uncertain knowledge and Mamdani inference to compensate the lack of data. Such anticipation of failure is also achieved in (Lee, 2002). They designed a new inference algorithm based on the generalized modus ponens (a fuzzy equivalent to modus ponens in classical logic) and syllogism. The knowledge is represented by fuzzy relations to recognize the type of accident, and thus allows operators to react faster. Holbert & Lin (2012) describes the development of the rule-based Fuzzy Logic Diagnostic Monitoring (FLDM). The proposed system is based on classical signal processing methods that are aggregated by fuzzy rules, and it has been applied on Nuclear Power Plant.

More particularly when using BBN, Zhao et al. (2020) uses fuzzy logic to discretize the sensors outputs in order to feed the BBN in a dynamic manner. The qualitative part of the dynamic BBN models the different variables considered for the system representation, their states and the static and dynamic relations between the variables. The authors state that when real valued signals are discretized, the frontier of the states cannot be clear and thus motivate the use of fuzzy sets.

These aspects may be useful for NARSIS WP5 “Supporting Tool for Severe Accident Management”.

5.6.3 General-purpose tools for developing fuzzy expert systems

In this section, we gather information about the main software programs for building fuzzy expert systems (Table 3). A more complete review of existing software programs can be found in(Alcalá-Fdez & Alonso, 2015).

Table 3 : Main fuzzy expert system softwares

Software name	Company	Type	License	Main functionalities
Qualicision	PSI	Software	Commercial	Decision automation KPI driven optimization
GUAJE	-	Software	Open source	Rule editor Rule learning Decision automation
FisPro	INRA	Software	Open source	Rule editor Rule learning Decision automation

Fuzzytech	INFORM GmbH	Software	Commercial	Rule editor Decision automation (embedded)
Fuzzy Tool box	Mathworks	Library	Commercial	Rule editor Decision automation
ExpressIF	CEA	Software	Commercial	Rule editor Rule learning (several techniques) Rule optimization Decision automation Access as a library, software or webservice

To the best of our knowledge, there is no specific software using fuzzy expert systems and specialized in monitoring of Nuclear Power Plants.

5.6.4 Summary

The afore-described approaches have shown that fuzzy logic and fuzzy expert systems are suitable for risk assessment and failure prevention in nuclear power plants.

In summary, the following benefits of using such approaches are outlined:

- Usually, safety data are either unavailable or unreliable : fuzzy logic and fuzzy expert system can combine knowledge and experience;
- This also allows to put a larger focus on the knowledge of 'domain experts', which is valuable to get insights into the mechanisms involved in nuclear power plants;
- The use of fuzzy logic enables a more realistic analysis thanks to the approximate reasoning and the use of no crisp information, which avoids threshold effects;
- A higher degree of flexibility is achieved through the use of linguistic variables, which avoids focusing on precise knowledge and does not need to be interpreted to be used;
- From a practical perspective, since each rule can be edited independently, it is seamless to update the rule base.

It should however be underlined that the nature of the safety analysis necessitates a framework that correctly handles uncertainty, because a large variety of different types of information sources are present within the different stages of a safety analysis (quantitative, qualitative/linguistic, probabilistic, interval-valued, etc.). Further developments based on the afore-outlined references are needed for this purpose.

6 Summary and recommendations

The state-of-the-art review in nuclear safety analysis have identified some key questions (Sect. 2.2) on the limitations the probabilistic setting for modelling the expert-based information, and on the elicitation procedure itself. In the view to improve the processing of such a specific source of information, we have followed a three-step approach:

1. Since the BBN-based approach is one major pillar of the NARSIS project, we have first analysed the feasibility of new approaches / procedures taking advantages (depending on the information available) of new uncertainty theories. Table 2 provides a detailed overview of the analysis. This has led to the following recommendations for BBN:

- The main challenge is the minimization of the elicitation workload on the experts owing to the large number of BBN parameters (CPT entries) while preserving the quality and consistency of the elicited result. A large number of different CPT elicitation procedures exist, but a consensus on the best practices is still lacking. Broader benchmark exercises are needed to cover a larger spectrum of methods (i.e. filling-up methods should be completed by Noisy- OR/MAX models, direct elicitation among others), of contexts (different network sizes, binary versus multivalued nodes, etc.), as well as of domains of application;
- Despite the clear advantages of BBN, it cannot be applied uncritically, because the probability values of such methods can only be considered approximations of the true probabilities and whatever the considered methods, they are all based on simplifications that may hamper the BBN performance. Here sensitivity methods (as developed in NARSIS WP3, task 3.2.2, see also an overview in Sect. 3.5) can play an important role;
- To both fulfil the objective of propagating the uncertainty and conducting the robustness analysis to the expert-based assumptions, the use of imprecise probabilities either relying on interval-valued probabilities within the setting of credal networks or within the Dempster-Shafer theory within the setting of evidential networks can be a valuable tool;
- Yet, though these new approaches enables an increase in expressiveness with respect to uncertainty representation, it should be underlined that this might come at the expense of higher complexity of the inference algorithms (and higher computational costs).

2. One option to constrain the uncertainties related to expert-based information is to combine the opinions of a panel of experts i.e. to combine several sources of information. Yet, a necessary preliminary step is to assess the “quality” of these sources of information, i.e. this raises the practical question of the evaluation of expert knowledge. In this view, we have investigated the benefit of using an alternative to the classical model of Cooke (1991) for evaluating expert-based information, i.e. an approach based on the possibility theory. This leads to the following recommendations for evaluation of expert-based information.

Regarding the problem of expert ranking,

- We show that the ranking can be completely different; especially for the calibration score in the investigated test cases. These differences should not be interpreted as a flaw in any of the approach: they are expected given the different focuses of each criterion. This can be considered a practical recommendation:
    - when the focus is on measuring the deviation from a reference value (i.e. a best estimate), the possibilistic calibration score is more appropriate;
    - when the focus is mainly on the statistical distribution of seed values in relation to the inter-percentiles given by the expert, the classical model is more appropriate.

- The ranking differences can be explained and searching the reasons for the dissimilarities can improve the evaluation analysis. Being aware that both approaches provide two viewpoints on the problem, it could envisage to mix these two pieces of information or leave it to the decision-maker to decide, which assessment best serves her/his particular interests.

Regarding the problem of prediction (and considering the results on our test cases),

- Whatever the approach, there is a benefit of relying on the result derived from the averaging of the expert-based information compared to relying on the information expert with best ranking score;
- Higher accuracy of the predictions is achieved by using the possibilistic approach, but this gain appears to come at the expense of higher imprecision (less informative);
- Using the classical model is however riskier, because we cannot be certain to find the true values within the interval derived from the averaging aggregation.

3. The third objective addresses the problem of expert-based knowledge modelling to support decision making. A solution based on fuzzy logic expert system tools was here investigated. On this basis, the following benefits of using such approaches are outlined for both risk assessment, and monitoring and diagnostic analysis of accidents of nuclear plants, namely:

- Usually, safety data are either unavailable or unreliable: fuzzy logic and fuzzy expert system can combine knowledge and experience. This also allows to put a larger focus on the knowledge of ‘domain experts’, which is valuable to get insights into the mechanisms involved in nuclear power plants;
- The use of fuzzy logic enables a more realistic analysis thanks to the approximate reasoning and the use of no crisp information, which avoids threshold effects;
- A higher degree of flexibility can be achieved through the use of linguistic variable, which avoids focusing on precise knowledge and does not need to be interpreted to be used;
- From a practical perspective, since each rule can be edited independently, it is seamless to update the rule base.

To conclude, the tools of new uncertainty theories can be considered valuable ingredients to support the safety analysis, and for NARSIS project in particular. They should however not be seen as supplements to “classical” probabilistic tools, but rather as complements to nuance the results relying on expert-based information, to put light on different perspectives, and to highlight potential flaws in the assessment process. Given the large variety of decision-making situations, finding a single appropriate framework appears to be debatable, and it is beneficial to take advantages of the strengths of multiple approaches to capture different types of information and knowledge important to inform the decision-making.

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